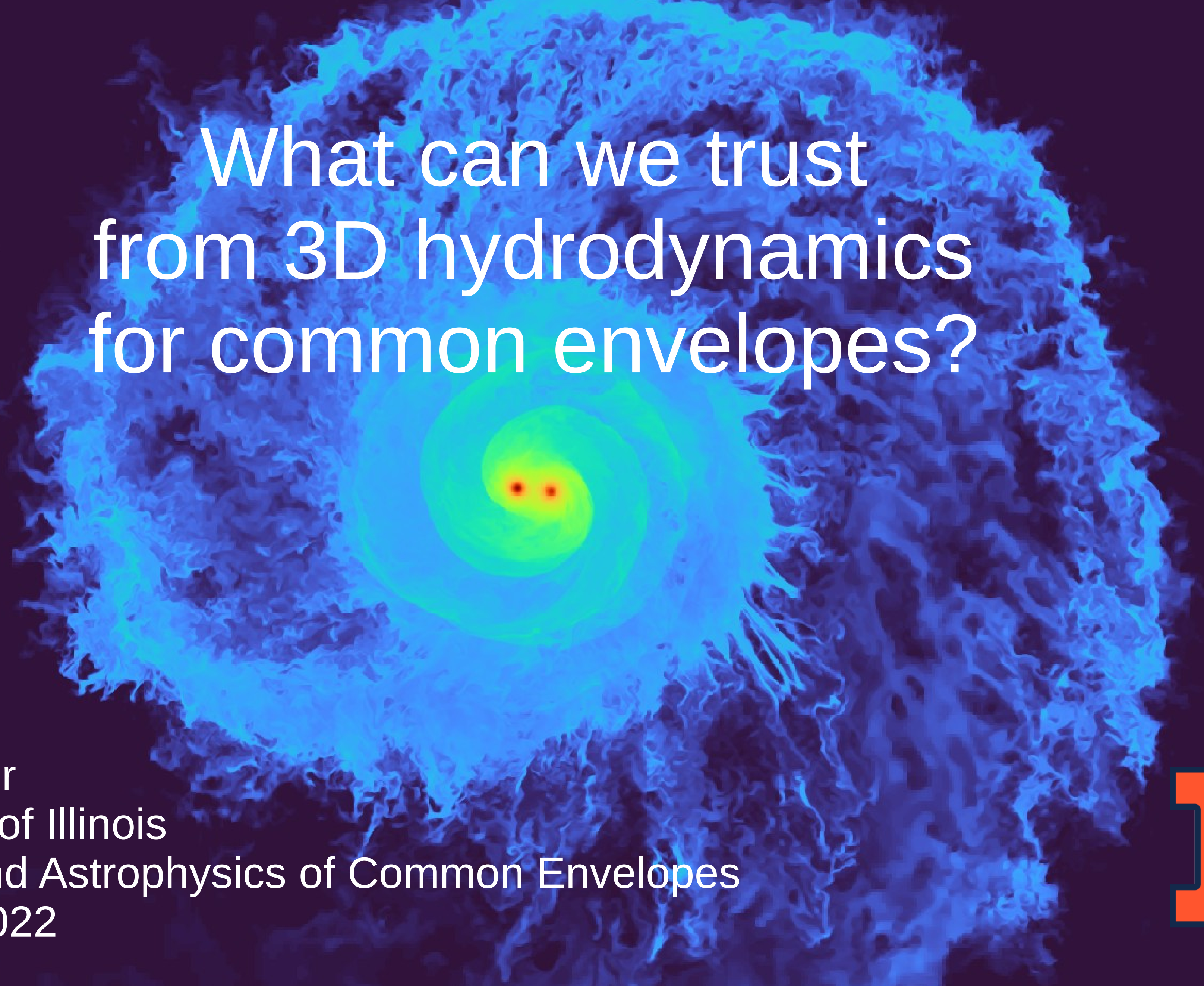

What can we trust from 3D hydrodynamics for common envelopes?

Paul Ricker
University of Illinois
Physics and Astrophysics of Common Envelopes
May 30, 2022

Outline


AAS | IOP Astronomy
Common Envelope Evolution
Natalia Ivanova
Stephen Justham
Paul Ricker
An American Astronomical Society and IOP Publishing partnership
AAS
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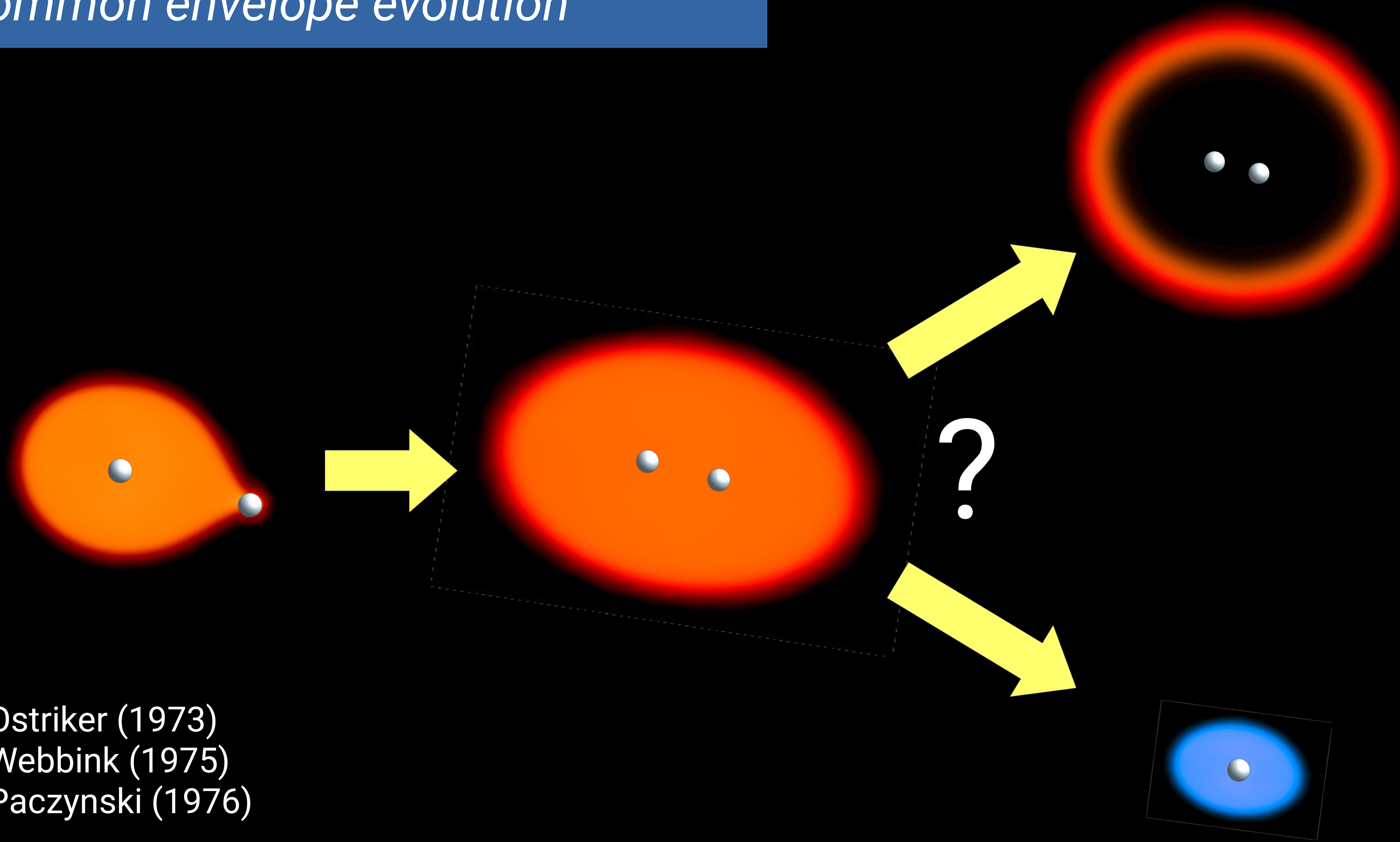
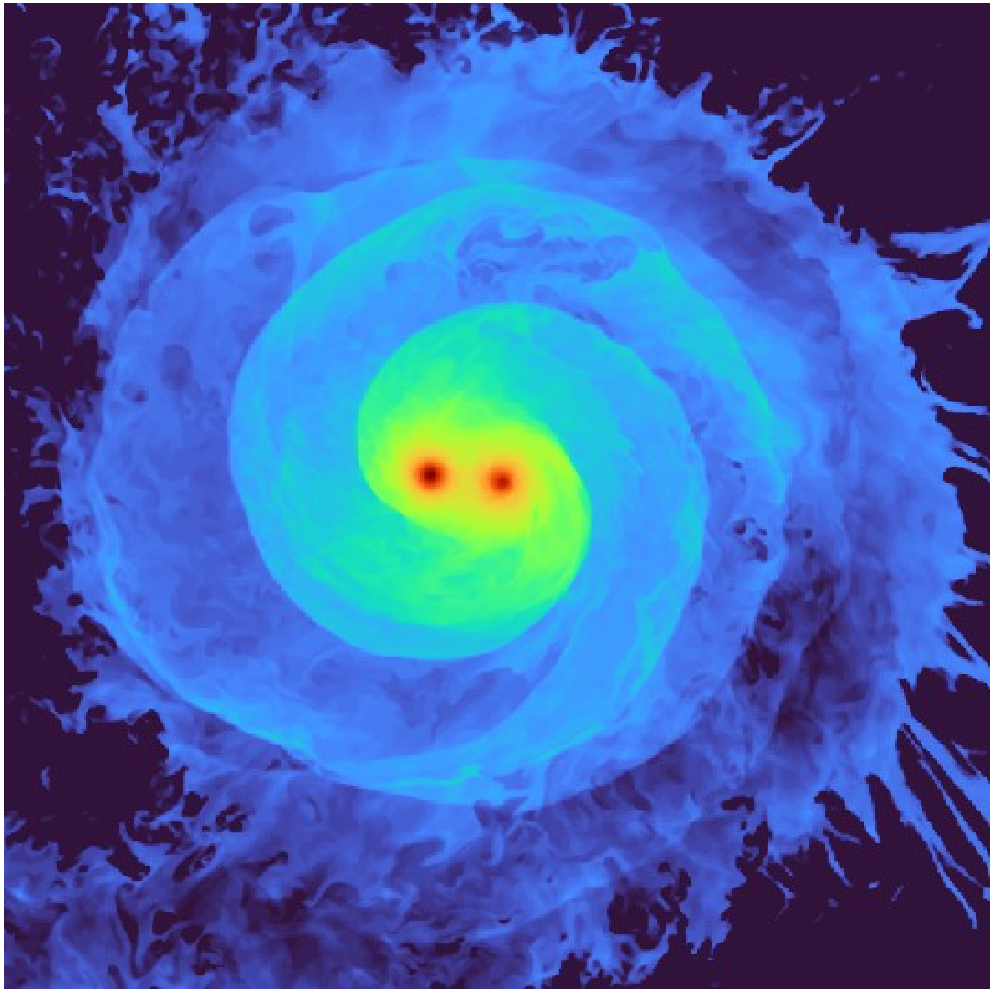
Common envelope evolution

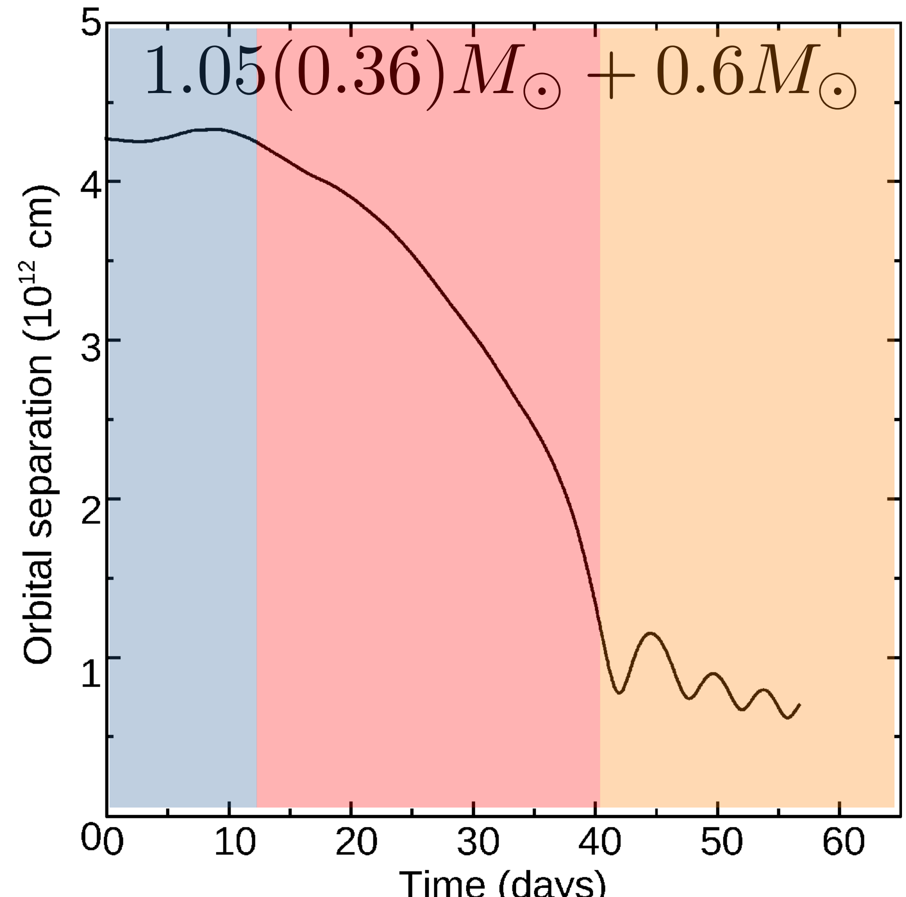


Ostriker (1973)
Webbink (1975)
Paczynski (1976)

A typical low-mass CE simulation



Ricker & Taam (2012)



Phases

- Loss of stability
- Dynamical plunge-in
- Slow spiral-in

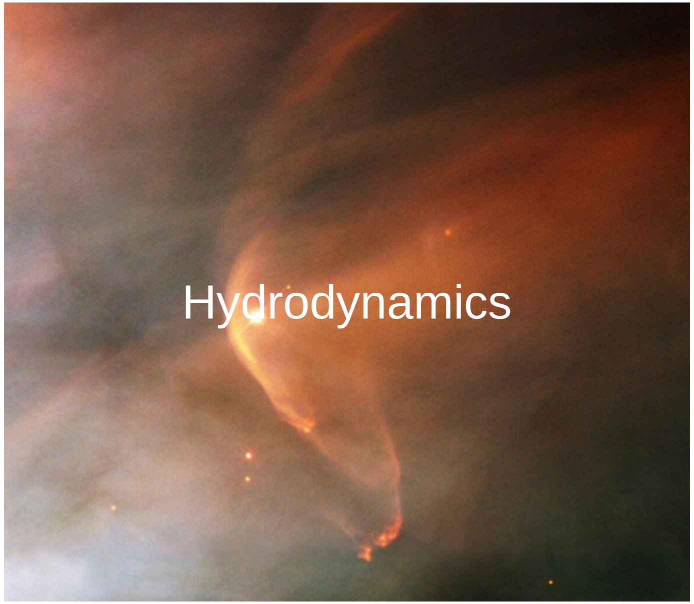
Outline

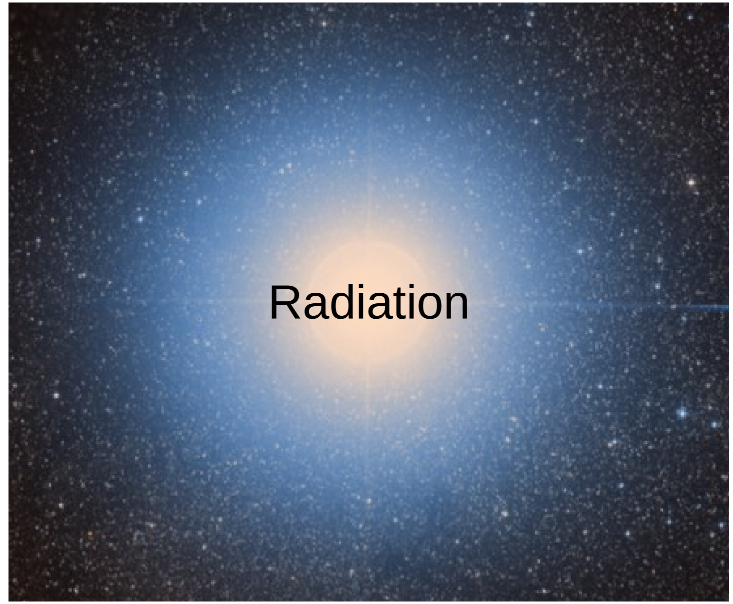
- Physics solved in 3D simulation codes
- Types of discretization
- Sources of error
- Conservation
- Initial and boundary conditions
- What can we trust?
- What do we need?

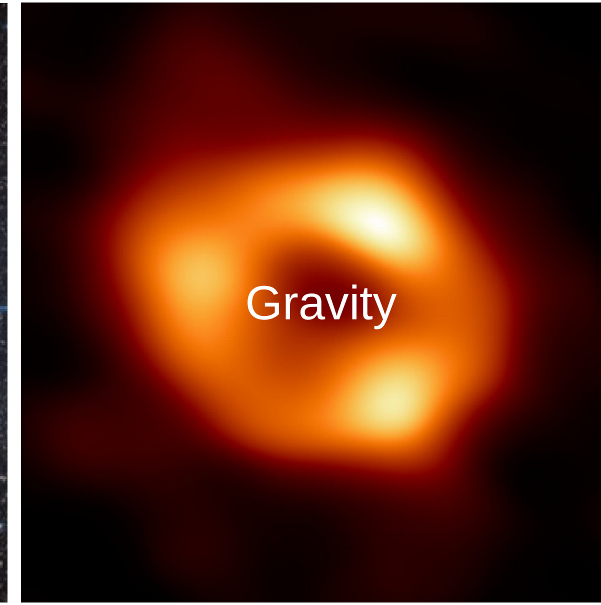
Outline

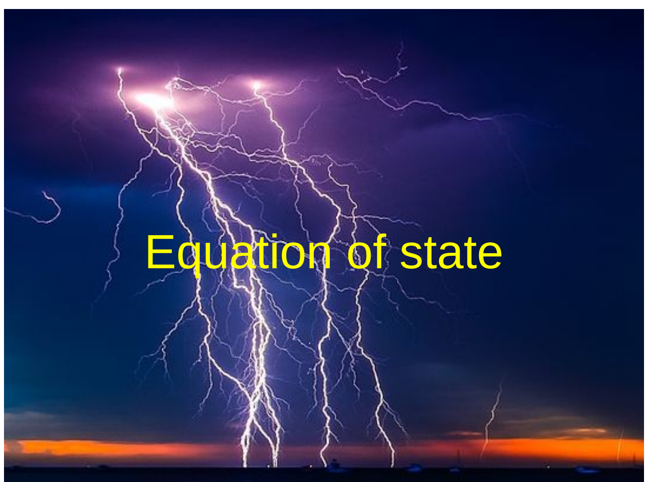
Physics

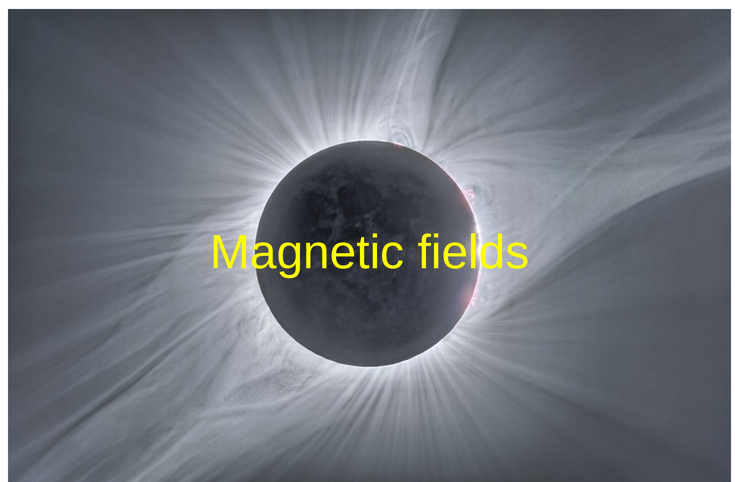
Physics included in 3D CE simulations

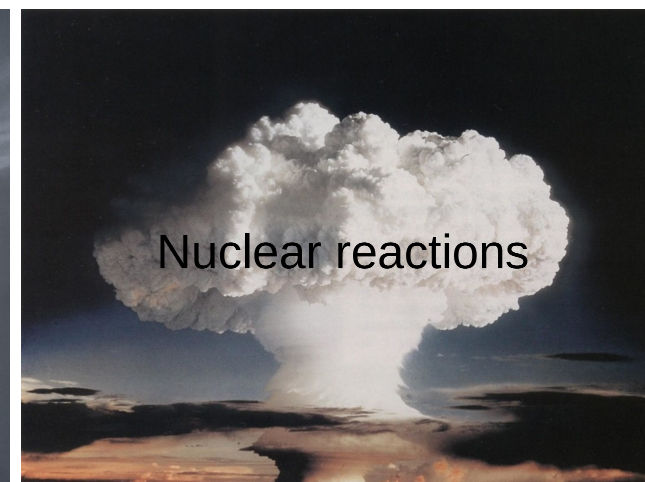












The equations that are solved – Eulerian form

$$\begin{aligned}\frac{\partial \rho X_i}{\partial t} + \nabla \cdot (\rho X_i \mathbf{v}) &= 0 \\ \frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v}\,\mathbf{v} + P\mathsf{I}) &= 0 \\ \frac{\partial \rho E}{\partial t} + \nabla \cdot [(\rho E + P)\mathbf{v}] &= 0\end{aligned}$$

$$E = \varepsilon + \frac{1}{2}v^2 \qquad \sum X_i = 1$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \mathbf{n}\cdot\nabla\right) I_\nu = \eta_\nu - \rho\kappa_\nu I_\nu$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} \mathrel{+}\approx \rho \mathbf{g}_{\mathrm{r}}(\rho, T, \mathbf{X}, \mathbf{v}, I_\nu)$$

$$\frac{\partial \rho E}{\partial t} \mathrel{+}\approx \rho \Gamma_{\mathrm{r}}(\rho, T, \mathbf{X}, \mathbf{v}, I_\nu)$$

$$\{\eta_\nu, \kappa_\nu\} \equiv \{\eta_\nu, \kappa_\nu\}(\rho, T, \mathbf{X}, \mathbf{v}, I_\nu)$$

$$\nabla^2 \phi = 4\pi G \rho$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} \mathrel{+}= -\rho \nabla \phi$$

$$\frac{\partial \rho E}{\partial t} \mathrel{+}= -\rho \mathbf{v} \cdot \nabla \phi$$

$$P \equiv P(\rho, \varepsilon, \mathbf{X})$$

$$T \equiv T(\rho, \varepsilon, \mathbf{X})$$

$$\begin{aligned}\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{v}) &= 0 \\ \nabla \cdot \mathbf{B} &= 0\end{aligned}$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} \mathrel{+}= \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}$$

$$\frac{\partial \rho X_i}{\partial t} \mathrel{+}= \rho \mathsf{R}_{ijk}(\rho, T) X_j X_k$$

$$\frac{\partial \rho \varepsilon}{\partial t} \mathrel{+}= \rho \sum \mathsf{Q}_{ijk}(\rho, T) X_j X_k$$

The equations that are solved – Eulerian form, equilibrium radiation diffusion

$$
\begin{aligned}
\frac{\partial \rho X_i}{\partial t} + \nabla\cdot(\rho X_i \mathbf{v}) &= 0\\
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla\cdot(\rho\mathbf{v}\,\mathbf{v} + P\mathsf{I}) &= 0\\
\frac{\partial \rho E}{\partial t} + \nabla\cdot[(\rho E + P)\mathbf{v}] &= 0
\end{aligned}
$$

$$E = \varepsilon + \frac{1}{2}v^2 \qquad \sum X_i = 1$$

$$E \rightarrow E + \frac{aT^4}{\rho}$$

$$P \rightarrow P + \frac{1}{3}aT^4$$

$$\frac{\partial \rho E}{\partial t} \mathrel{+}\approx \nabla\cdot\left(\frac{4ac}{3\rho\bar{\kappa}}T^3\nabla T\right)$$

$$\bar{\kappa} \equiv \bar{\kappa}(\rho, T, \mathbf{X})$$

$$\nabla^2\phi = 4\pi G\rho$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} \mathrel{+}= -\rho\nabla\phi$$

$$\frac{\partial \rho E}{\partial t} \mathrel{+}= -\rho\mathbf{v}\cdot\nabla\phi$$

$$P \equiv P(\rho, \varepsilon, \mathbf{X})$$

$$T \equiv T(\rho, \varepsilon, \mathbf{X})$$

$$
\begin{aligned}
\frac{\partial \mathbf{B}}{\partial t} + \nabla\times(\mathbf{B}\times\mathbf{v}) &= 0\\
\nabla\cdot\mathbf{B} &= 0
\end{aligned}
$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} \mathrel{+}= \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}$$

$$\frac{\partial \rho X_i}{\partial t} \mathrel{+}= \rho\mathsf{R}_{ijk}(\rho, T)X_j X_k$$

$$\frac{\partial \rho \varepsilon}{\partial t} \mathrel{+}= \rho\sum \mathsf{Q}_{ijk}(\rho, T)X_j X_k$$

The equations that are solved – Lagrangian form, equilibrium radiation diffusion

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\mathbf{v}$$

$$\frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho}\nabla P \qquad \frac{DX_i}{Dt} = 0$$

$$\frac{D\varepsilon}{Dt} = -\frac{P}{\rho}\nabla\cdot\mathbf{v}$$

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla$$

$$\varepsilon \rightarrow \varepsilon + \frac{aT^4}{\rho}$$

$$P \rightarrow P + \frac{1}{3}aT^4$$

$$\frac{D\varepsilon}{Dt} \mathrel{+}\approx \frac{1}{\rho}\nabla\cdot\left(\frac{4ac}{3\rho\bar{\kappa}}T^3\nabla T\right)$$

$$\bar{\kappa} \equiv \bar{\kappa}(\rho, T, \mathbf{X})$$

$$\nabla^2\phi = 4\pi G\rho$$

$$\frac{D\mathbf{v}}{Dt} \mathrel{+}= -\nabla\phi$$

$$P \equiv P(\rho, \varepsilon, \mathbf{X})$$

$$T \equiv T(\rho, \varepsilon, \mathbf{X})$$

$$\frac{D\mathbf{B}}{Dt} = -\mathbf{B}(\nabla\cdot\mathbf{v}) + (\mathbf{B}\cdot\nabla)\mathbf{v}$$

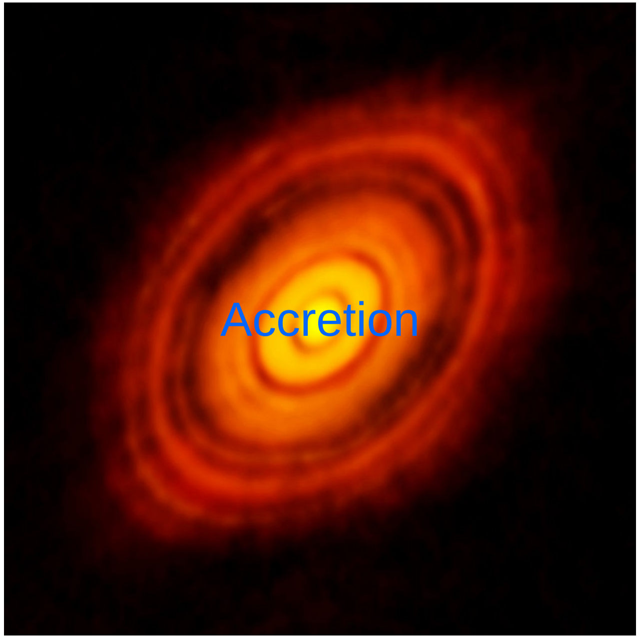
$$\nabla\cdot\mathbf{B} = 0$$

$$\frac{D\mathbf{v}}{Dt} \mathrel{+}= \frac{1}{4\pi\rho}(\nabla\times\mathbf{B})\times\mathbf{B}$$

$$\frac{DX_i}{Dt} \mathrel{+}= \mathsf{R}_{ijk}(\rho, T)X_jX_k$$

$$\frac{D\varepsilon}{Dt} \mathrel{+}= \sum \mathsf{Q}_{ijk}(\rho, T)X_jX_k$$

Technically not separate physics



Large eddy simulation (LES):

Directly solve for large-scale motions

Add terms to momentum and energy equations to account for correlations on filtered-out small scales

Terms based on a model for the small scales

Sink particle techniques:

Treat accretor as a particle moving in gravity field

Add to mass/spin/etc by estimating accretion rate from resolved gas

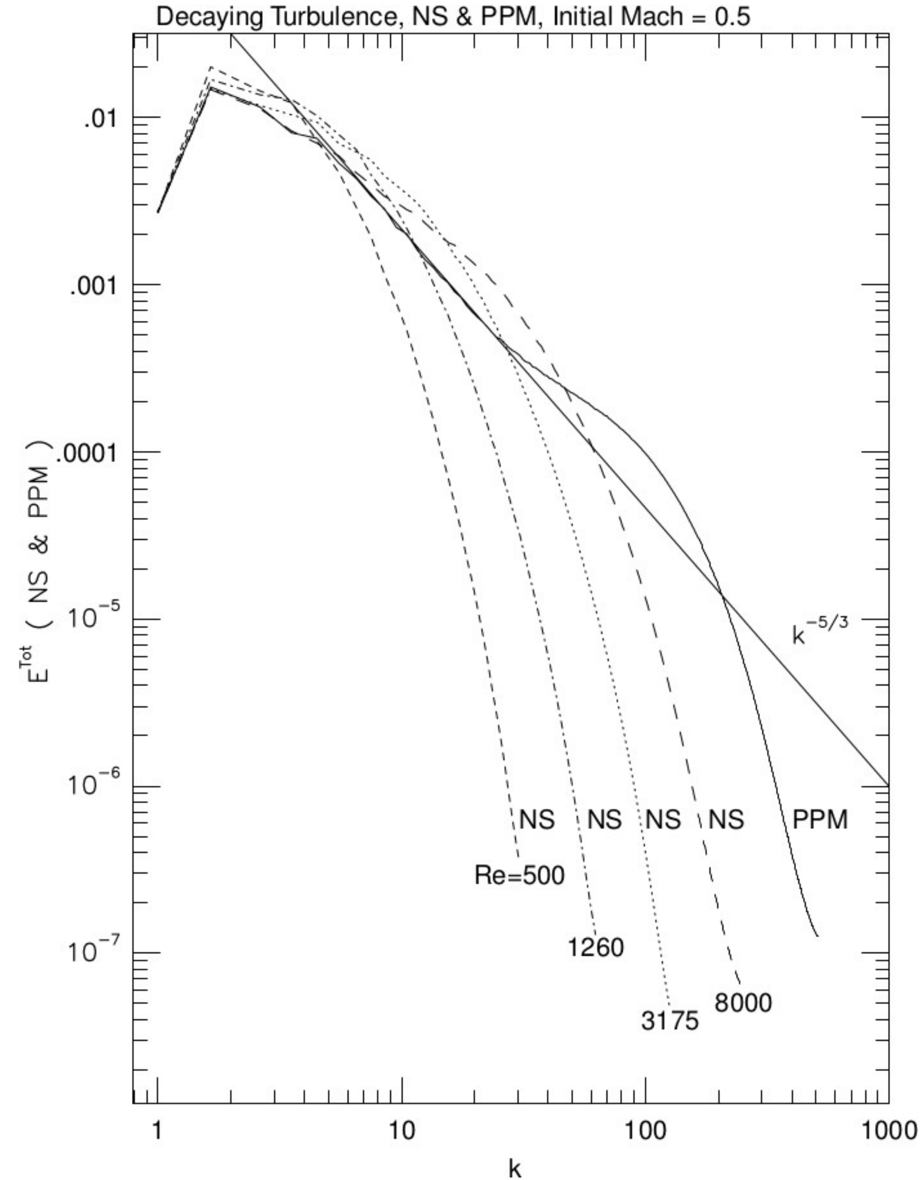
Remove amounts from resolved gas and apply feedback

Dissipation and turbulence

Most astrophysical simulations are "implicit LES"

May be OK for some things (energy dissipation), not others (backscatter)

Particularly a concern for MHD



Sytine et al. (2000)

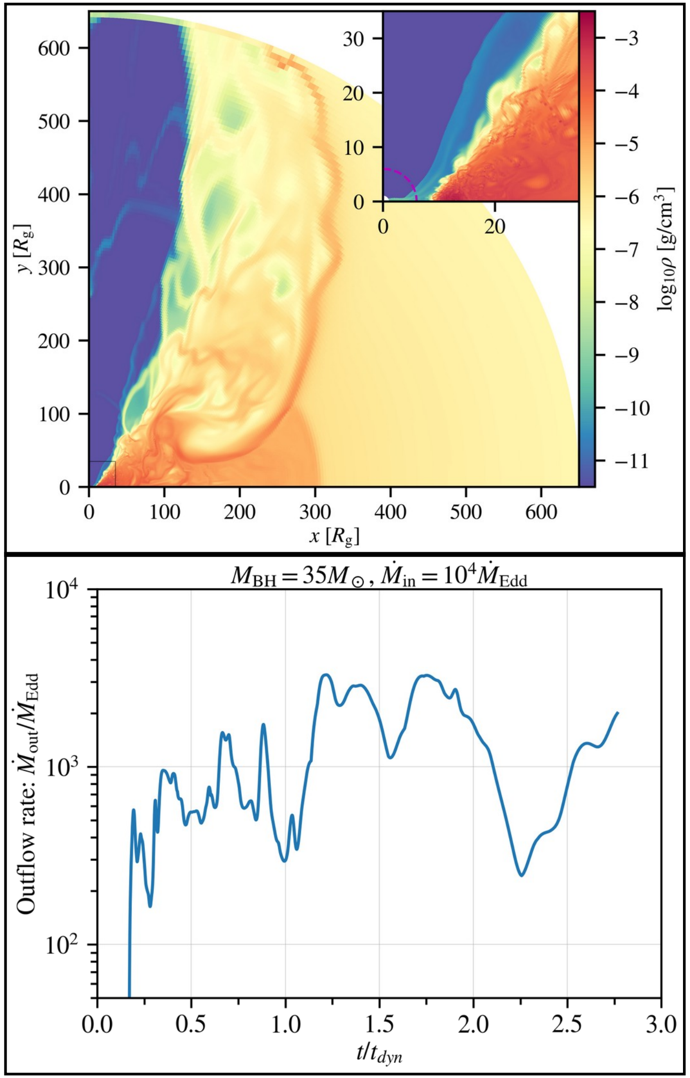
Assumptions we typically make

- Local thermodynamic equilibrium (LTE) between gas and radiation
- ~~Inviscid, ideal HD/MHD~~ viscosity/conductivity/resistivity from truncation error
- ~~Gamma-law EOS~~ fortunately not so much anymore
- Nuclear reactions aren't important
- Core expansion/contraction isn't important
- Jets *and radiation* from compact companions aren't important (Fragile talk)
- Radiation transport more generally can be neglected (Hungerford talk)
- And if it’s not, continuum opacity dominates, no line-driven winds
- And if it’s not, it's ok if we can't resolve the photosphere
- Dust and dust chemistry are typically neglected (Mauney talk)

Simulations solve a restricted set of physics whose applicability must be demonstrated

Super-Eddington accretion onto compact companions

- **COSMOS++** (Anninos et al. 2003)
  - General relativistic (magneto)hydrodynamics
  - Neutrino cooling and nuclear burning
- Black holes in 2D
  - Highly variable outflows
- Neutron stars in 1D
  - Evolving accretion shock
  - Nucleosynthesis at NS surface



Holgado et al. (in prep)

Super-Eddington accretion onto compact companions

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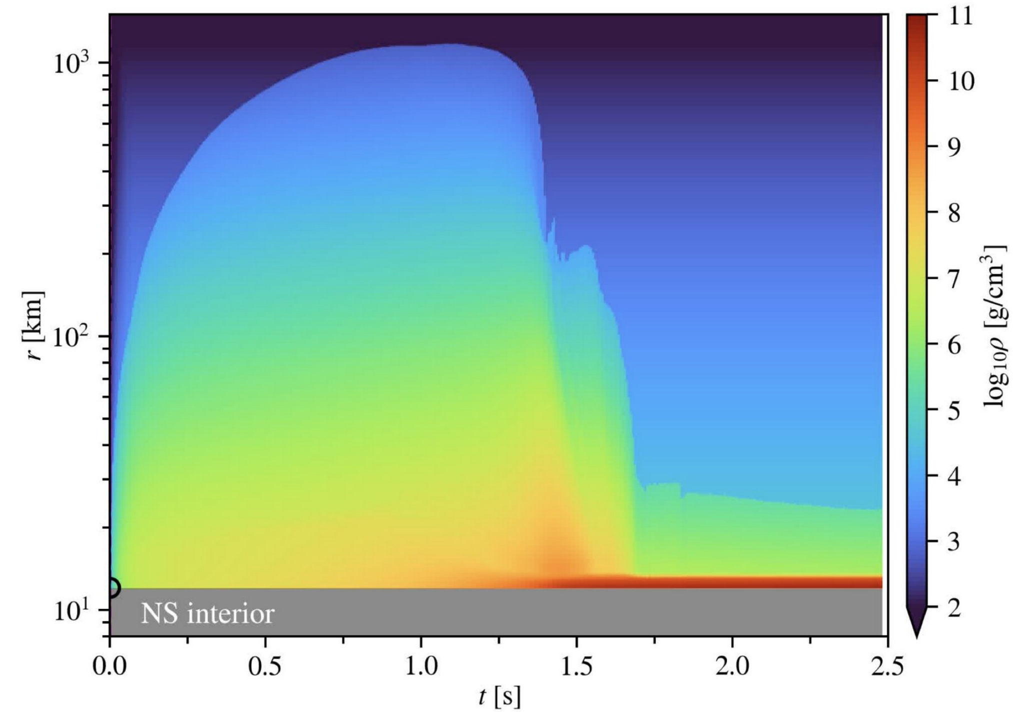
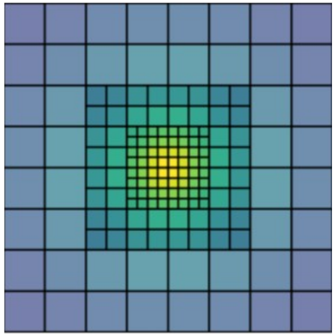


Better accretion/feedback models are needed

Holgado et al. (in prep)

Codes

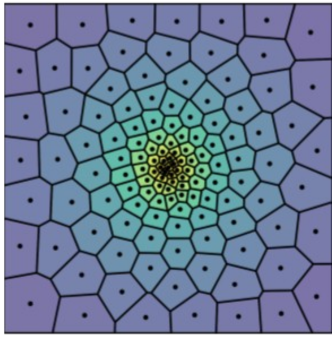
Types of discretization



Structured adaptive mesh refinement (AMR)

Non-overlapping elements, finite volume discretization
Eulerian frame of reference

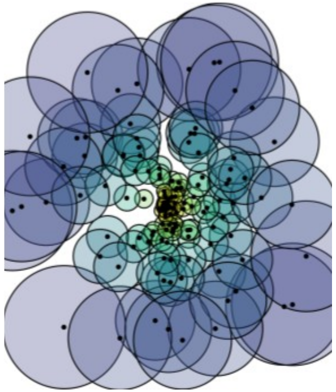
Examples: FLASH, Enzo, AstroBEAR, Athena++



Moving mesh (MM) / unstructured mesh

Non-overlapping elements, finite volume or element discretization
Eulerian or Lagrangian frame of reference

Examples: Arepo, Manga



Smoothed particle hydrodynamics (SPH)

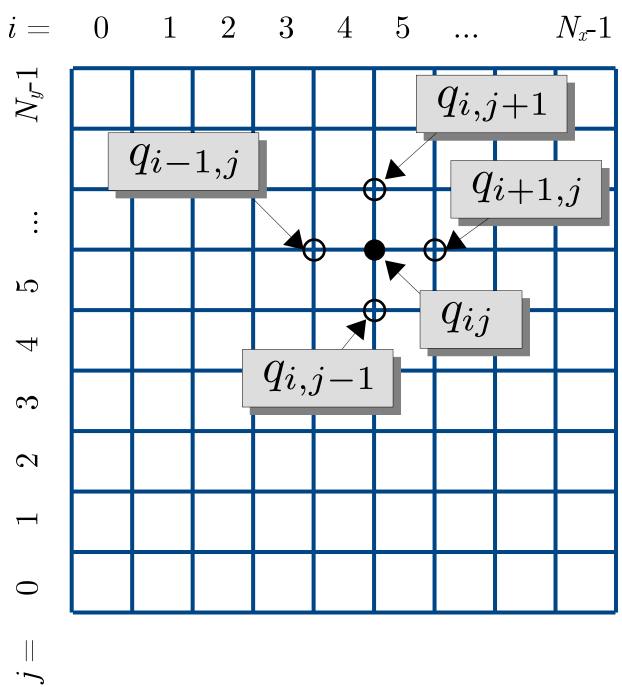
Overlapping elements, finite difference discretization
Lagrangian frame of reference

Examples: StarSmasher, Phantom, Gadget, SNSPH

What numbers get stored?

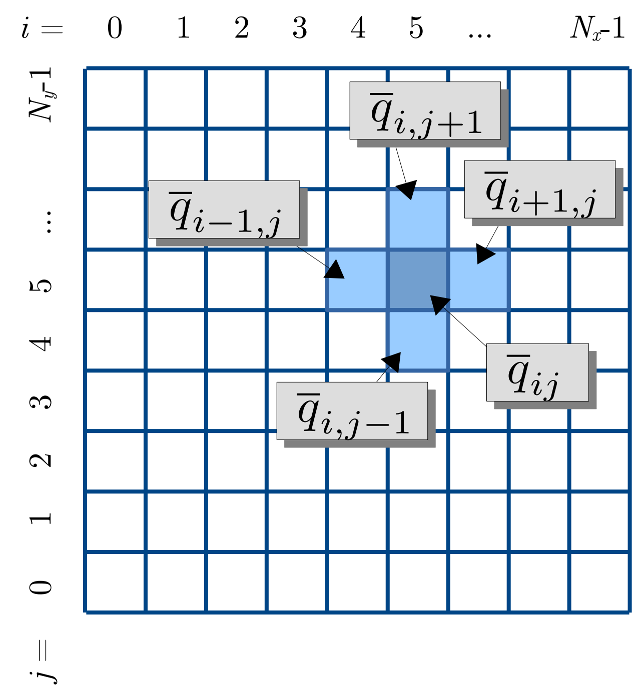
Finite differencing
Function values at points

$$q_{ij} \equiv q(x_i, y_j)$$



Finite volume
Function averages over zones

$$\overline{q}_{ij} \equiv \frac{1}{\Delta x \Delta y} \int_{x_{i-1/2}}^{x_{i+1/2}} \int_{y_{j-1/2}}^{y_{j+1/2}} q(x, y)\, dx\, dy$$

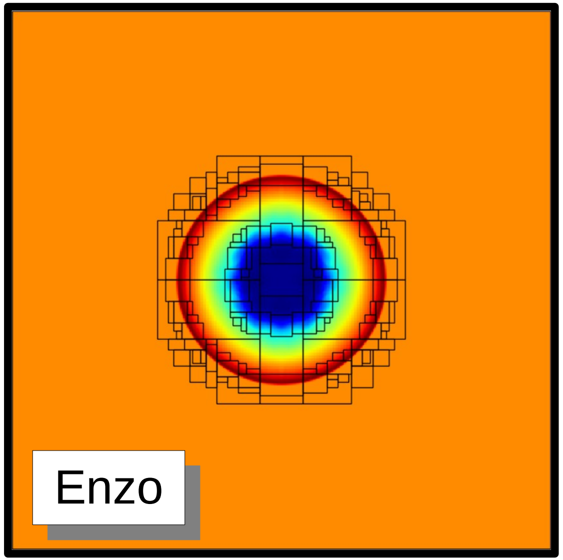


Adaptive mesh refinement (AMR)

Patch-based AMR (Berger & Oliger 1984)

Evolve solution on a hierarchy of mesh **patches**
Each patch can have different number of zones
Factor of 2 or 4 refinement between levels

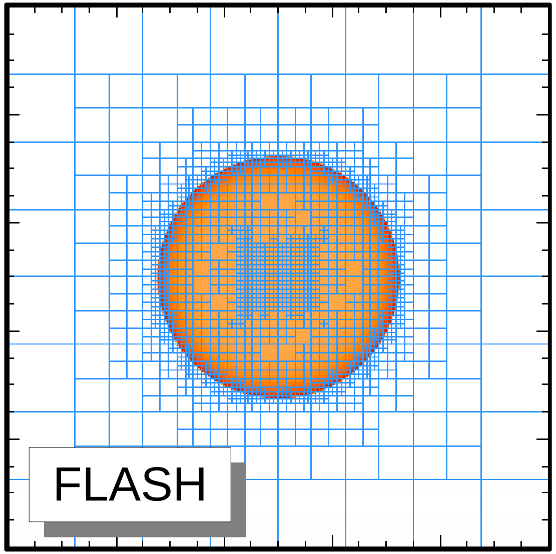
Benefit: more flexible



Oct-tree AMR (Quirk 1991)

Evolve solution on a hierarchy of mesh **blocks**
Each block has same number of zones
Factor of 2 refinement between levels

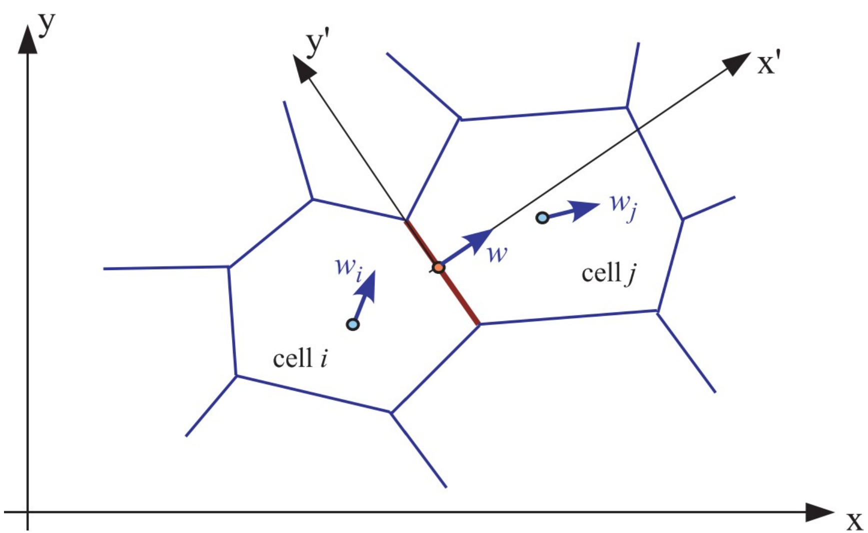
Benefits: more cache-efficient, parallel scaling

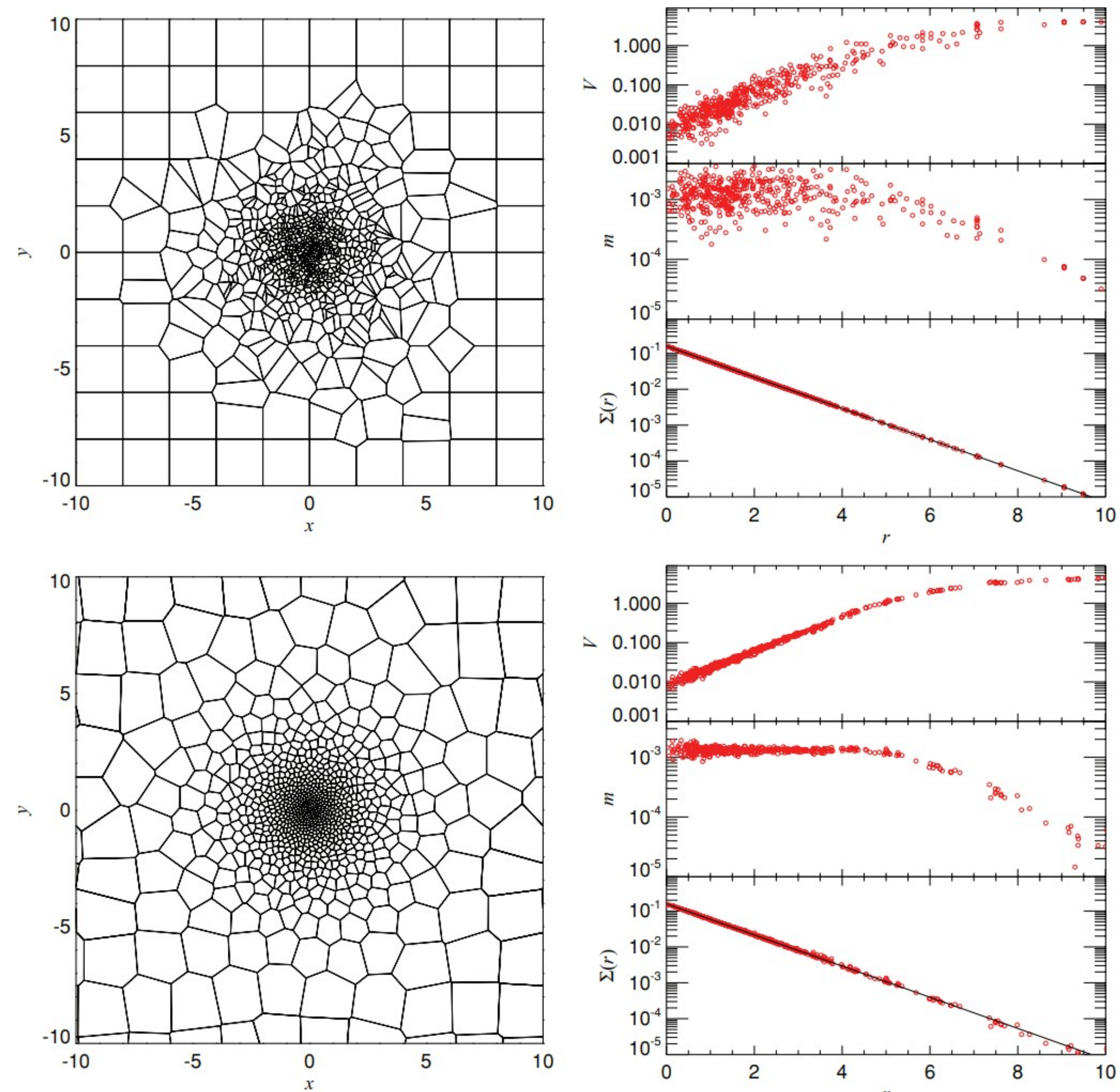


Moving/unstructured meshes

Particles move with local fluid velocity, used to generate unstructured mesh using regularized Voronoi tesselation

Finite volume differencing of equations of motion





Springel (2010)

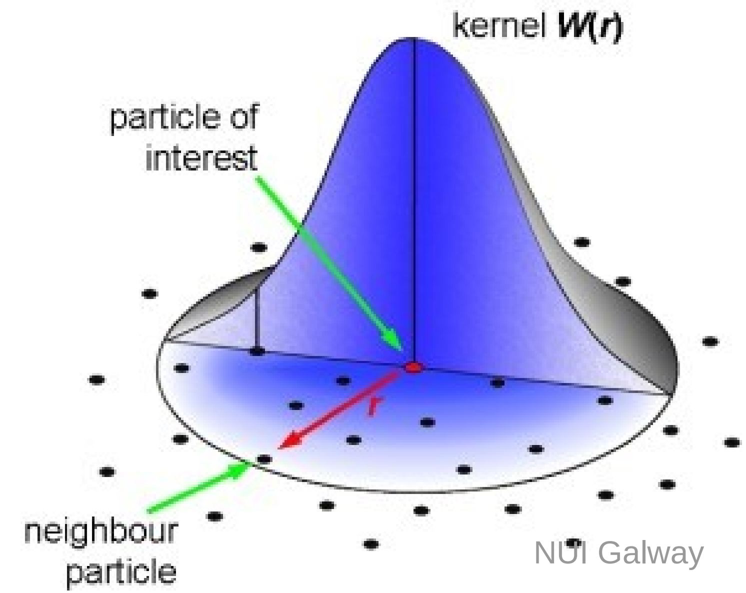
Smoothed particle hydrodynamics

Each particle i tagged with fluid quantities A_i:

$$A(\mathbf{x}) \equiv \sum_i m_i \frac{A_i}{\rho_i} W(\mathbf{x}-\mathbf{x}_i, h_i)$$

Kernel

Softening length



Equations of motion:

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i$$

$$\frac{d\mathbf{v}_i}{dt} = -\sum_{j\neq i} m_i \left(\frac{P_i}{\Omega_i \rho_i^2} \nabla W_{ij}(h_i) - \frac{m_j^2}{m_i^2} \frac{P_j}{\Omega_j \rho_j^2} \nabla W_{ij}(h_j) \right) - \mathbf{g}_i$$

Accounts for varying h

Outline

Erorrs

Sources of error

Errors in an individual calculation (single zone or particle)

Truncation error

All discretized versions of ODEs/PDEs involve truncation of infinite Taylor series.

$$q(x) = q(x_i) + q'(x_i)(x - x_i) + \frac{1}{2}q''(x_i)(x - x_i)^2$$

A third-order approximation: error is $\mathcal{O}(\Delta x^3)$

Roundoff error

All floating-point computations involve truncated infinite binary representations of numbers.

Fraction	Single Precision Decimal	Binary 32-bit IEEE representation	Cast to Double Precision Decimal
1/3	0.333333	00111110101010101010101010101011	0.3333329856395721 4
1/10	0.1	00111101110011001100110011001101	0.1000000014901161 2
1/2	0.5	00111111000000000000000000000000	0.5

Sources of error

Accumulation of errors

Truncation error

Accumulates partially – depends on the nature of the fluid flow

Higher order is usually better…. But:

- Larger differencing stencil → more parallel communication
- Numerical methods harder to construct

Roundoff error

Always accumulates with increasing number of steps

Minimizing requires:

- Double precision arithmetic
- Higher order time integration

Linear advection

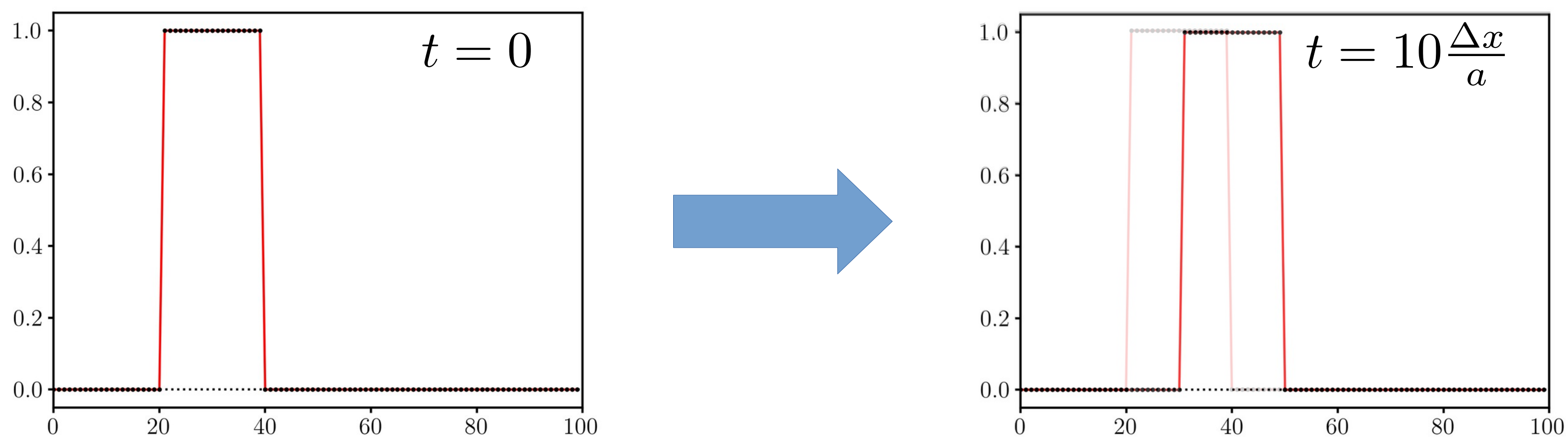
Continuity equation with constant speed a is called the **linear advection equation**:

$$\frac{\partial q}{\partial t} + a\frac{\partial q}{\partial x} = 0 \qquad a = \text{constant}$$

Easy to solve: let $q(x,t) \equiv q(s)$ where $s \equiv x - at$

$$\frac{\partial q}{\partial t} = \frac{dq}{ds}\frac{\partial s}{\partial t} = -a\frac{dq}{ds} \qquad \frac{\partial q}{\partial x} = \frac{dq}{ds}\frac{\partial s}{\partial x} = \frac{dq}{ds} \qquad \Longrightarrow \qquad \frac{\partial q}{\partial t} + a\frac{\partial q}{\partial x} = 0$$

Any initial shape just moves with speed a.



Linear advection

Continuity equation with constant speed a is called the **linear advection equation**:

$$\frac{\partial q}{\partial t} + a\frac{\partial q}{\partial x} = 0 \qquad a = \text{constant}$$

Two classic finite difference schemes (let $a > 0$):

$$q_i^{n+1} = q_i^n - \frac{a\Delta t}{\Delta x}\left(q_i^n - q_{i-1}^n\right)$$

Upwind scheme (Courant 1952)

$$q_i^{n+1} = q_i^n - \frac{a\Delta t}{2\Delta x}(q_{i+1}^n - q_{i-1}^n) + \frac{a^2\Delta t^2}{2\Delta x^2}(q_{i+1}^n - 2q_i^n + q_{i-1}^n)$$

Lax-Wendroff scheme (1960)

Both **conditionally stable**: require **Courant-Friedrichs-Lewy (CFL)** criterion

$$|a|\frac{\Delta t}{\Delta x} < 1$$

Physical behavior depends on leading order of truncation error (odd vs. even)

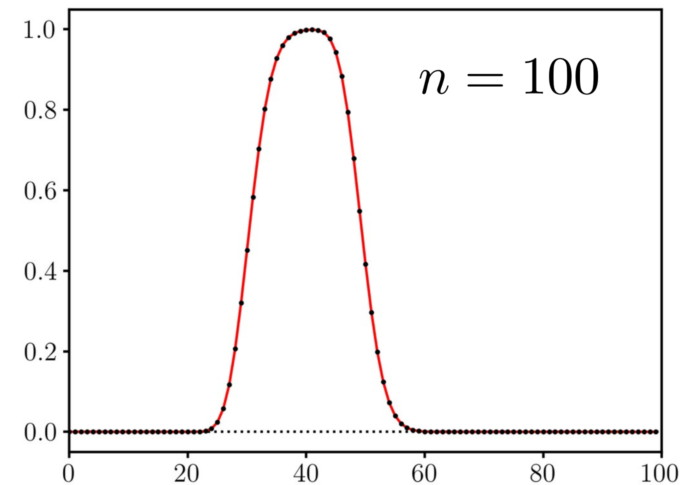
Odd: diffusive

Upwind is equivalent to solving

$$\boxed{\frac{\partial q}{\partial t} = -a\frac{\partial q}{\partial x}} + \boxed{\frac{1}{2}a\Delta x\frac{\partial^2 q}{\partial x^2}} + \dots$$

Our PDE

Error looks like a diffusion term!



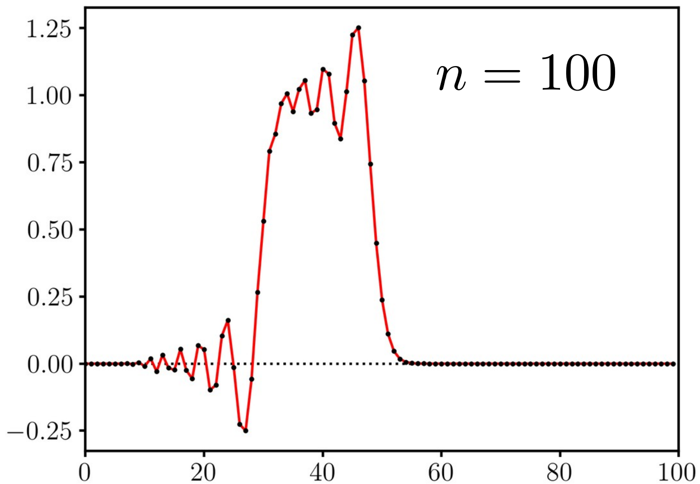
Even: dispersive

Lax-Wendroff is equivalent to solving

$$\boxed{\frac{\partial q}{\partial t} = -a\frac{\partial q}{\partial x}} + \boxed{\frac{1}{6}a(\Delta x^2 - a^2\Delta t^2)\frac{\partial^3 q}{\partial x^3}} + \dots$$

Our PDE

Error looks like an advection term with wavenumber-dependent speed!



Demonstrating convergence

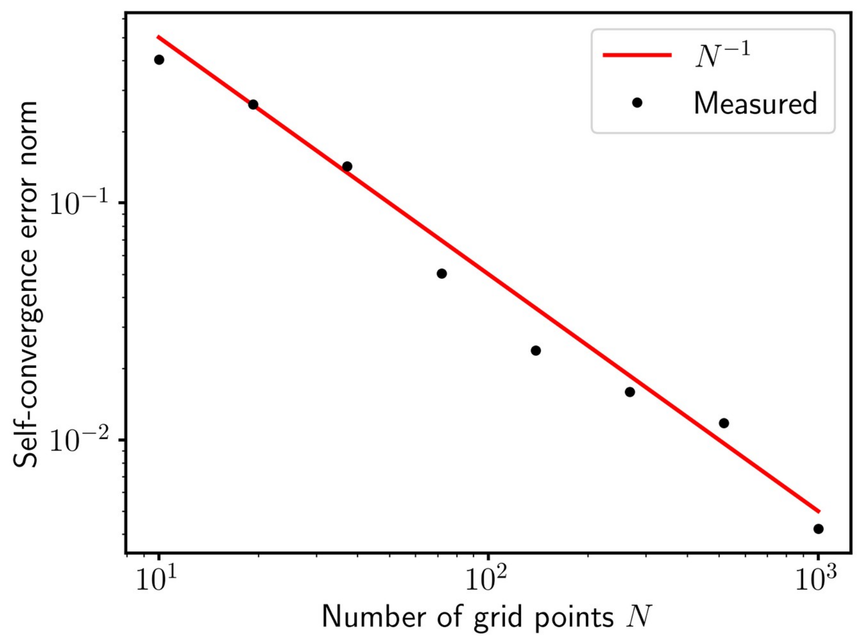
Global errors – measure using **norms**:

$$\mathrm{L}_n \ \text{norm} \equiv \left[\frac{1}{N} \sum_{i=0}^{N-1} \left| q_i^{\text{numerical}} - q_i^{\text{analytical}} \right|^n \right]^{1/n}$$

Frequently used: L_1, L_2, or L_∞:

$$\mathrm{L}_\infty \ \text{norm} \equiv \lim_{n \to \infty} \mathrm{L}_n \ \text{norm} = \sup \left| q_i^{\text{numerical}} - q_i^{\text{analytical}} \right|$$

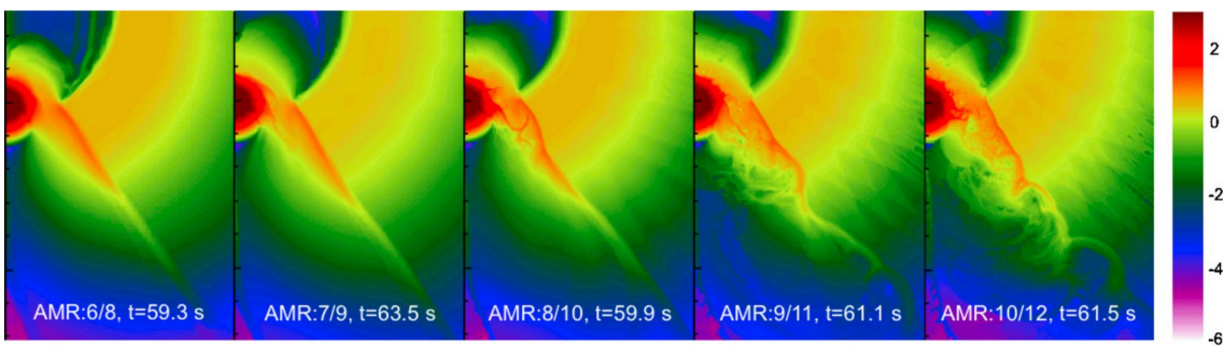
No analytical solution?
Try self-convergence
– not the same as convergence!

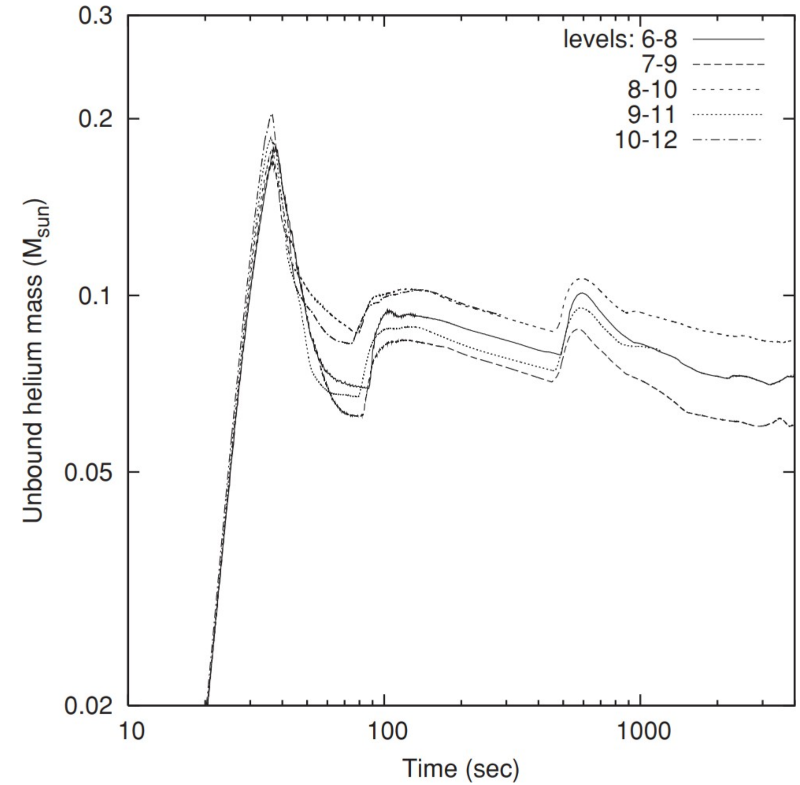


Demonstrating convergence

Pan, Ricker, & Taam (2010) – SNIa in He+WD system

- Don't expect normed convergence to a fixed solution for turbulent flows
- Measure desired quantities at different resolutions
- Numerical transition to turbulence can limit convergence of quantities that depend sensitively on mixing



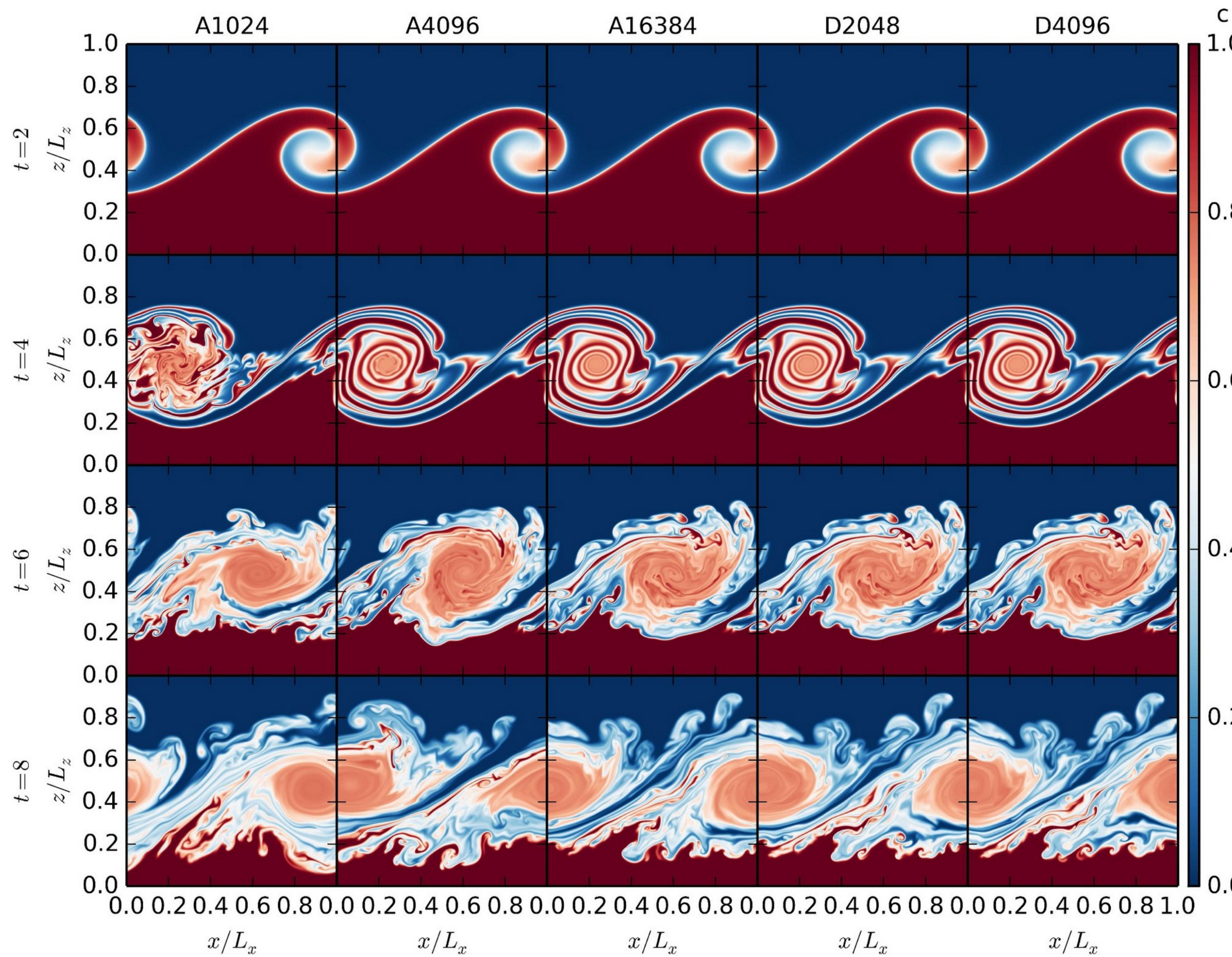


Numerical self-convergence must be demonstrated for each simulation

Artifacts

Lecoanet et al. (2016) – comparison of Athena and DEDALUS on Kelvin-Helmholtz instability problem

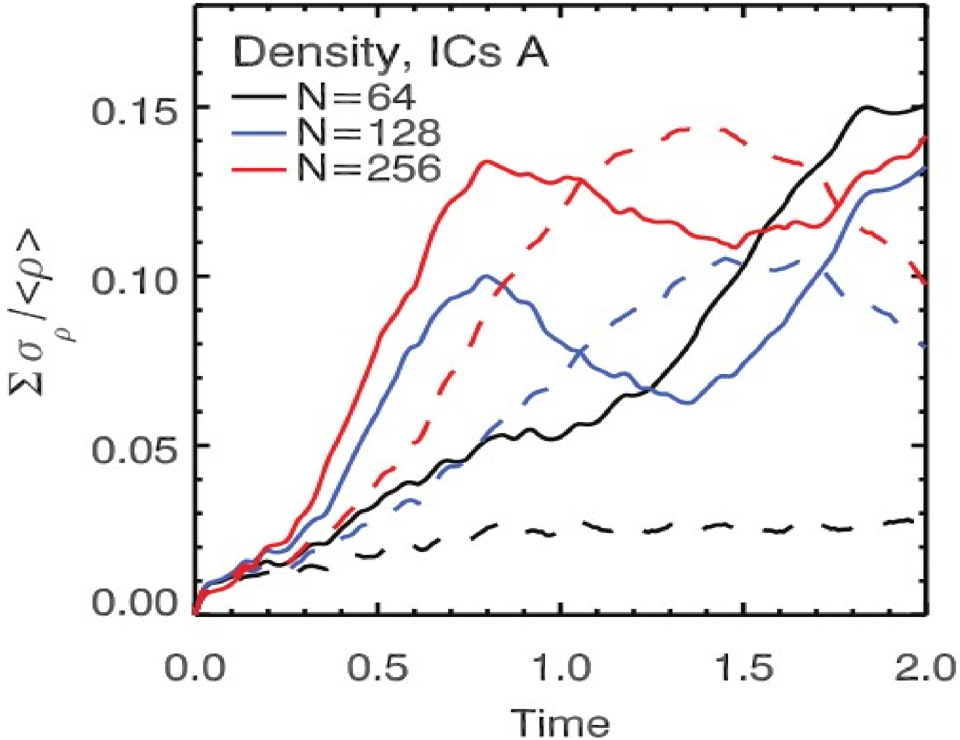
Low-resolution Athena run numerically seeds physical instabilities that do not arise at higher resolution



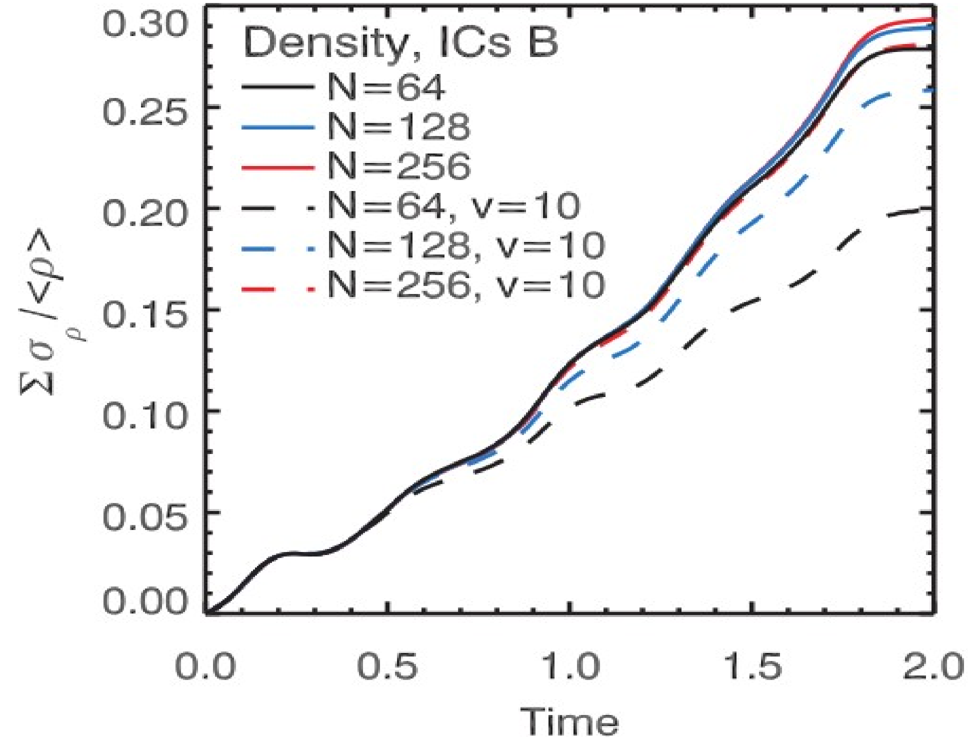
Confidence that we have converged to a correct result requires code comparisons, especially when physics beyond hydrodynamics is included

Galilean invariance

Velocity-dependent diffusivity breaks Galilean invariance in AMR codes.



Sharp initial interface



Smooth initial interface

Robertson et al. (2010) – ART simulations of KH instability w/ & w/o background Mach 10 flow

Adequate resolution of initial material surfaces is needed for convergence

Outline

Conservation

Finite volume

Frequently need to satisfy integral constraints. Let $q =$ mass density ρ :

$$\int_0^L dx \left[\frac{\partial \rho}{\partial t} + a\frac{\partial \rho}{\partial x}\right] = 0 \qquad \Longrightarrow \qquad \frac{d}{dt}\int_0^L \rho\, dx \equiv \frac{dM}{dt} = -a[\rho(L) - \rho(0)]$$

Finite volume defines

$$\overline{q}_i^n \equiv \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} q(x,t)\, dx$$

Consider the generalized **scalar advection equation**:

$$\frac{\partial q}{\partial t} + \frac{\partial f(q)}{\partial x} = 0$$

f is the **flux function**

Integrate over a zone:

$$\frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} dx \left[\frac{\partial q}{\partial t} + \frac{\partial f(q)}{\partial x}\right] = 0$$

$$\frac{d\overline{q}_i}{dt} + \frac{f(q_{i+1/2}) - f(q_{i-1/2})}{\Delta x} = 0 \qquad\qquad q_{i+1/2} \equiv q(x_{i+1/2})$$

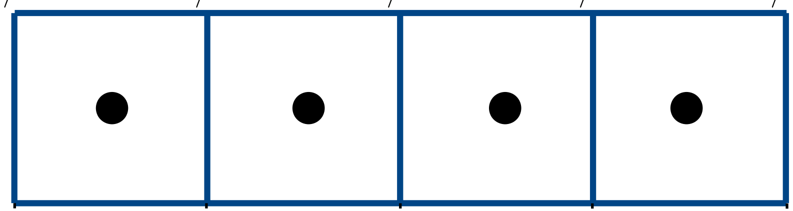
Finite volume

Now average over timestep:

$$\frac{1}{\Delta t}\int_{t_n}^{t_{n+1}} dt : \qquad \frac{\overline{q}_i^{n+1}-\overline{q}_i^{n}}{\Delta t}+\frac{\overline{f}_{i+1/2}^{n+1/2}-\overline{f}_{i-1/2}^{n+1/2}}{\Delta x}=0$$

Exact equation!

Same thing is added and subtracted from each pair of zones:



$$\frac{\overline{q}_0^{n+1}-\overline{q}_0^{n}}{\Delta t}=\frac{\overline{f}_{-1/2}}{\Delta x}-\frac{\overline{f}_{1/2}}{\Delta x}$$

$$\frac{\overline{q}_1^{n+1}-\overline{q}_1^{n}}{\Delta t}=\frac{\overline{f}_{1/2}}{\Delta x}-\frac{\overline{f}_{3/2}}{\Delta x}$$

$$\frac{\overline{q}_2^{n+1}-\overline{q}_2^{n}}{\Delta t}=\frac{\overline{f}_{3/2}}{\Delta x}-\frac{\overline{f}_{5/2}}{\Delta x}$$

$$\frac{\overline{q}_3^{n+1}-\overline{q}_3^{n}}{\Delta t}=\frac{\overline{f}_{5/2}}{\Delta x}-\frac{\overline{f}_{7/2}}{\Delta x}$$

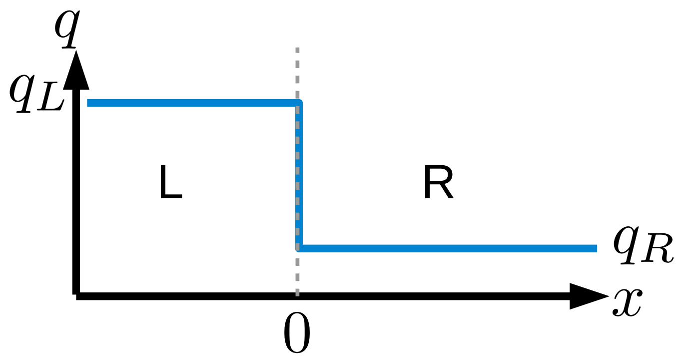
$$\overline{f}_{i+1/2}^{n+1/2}\equiv\underbrace{\frac{1}{\Delta t}\int_{t_n}^{t_{n+1}} f\left(q_{i+1/2}\right)dt}$$

Approximate with some function of known q's

Time and space accuracy depends on how the fluxes ($\overline{f}$) are **reconstructed** from solution data (q)

Why is conservation important?

Riemann problem for nonlinear scalar advection equation:



$$\frac{\partial q}{\partial t} + \frac{\partial f(q)}{\partial x} = \frac{\partial q}{\partial t} + \frac{df}{dq}\frac{\partial q}{\partial x} = 0$$

wave speed

Self-similar solutions: $q(x,t) \equiv w(x/t)$

$$\frac{\partial q}{\partial t} = -w' \frac{x}{t^2} \qquad \frac{\partial q}{\partial x} = \frac{w'}{t}$$

$$\frac{\partial q}{\partial t} + \frac{df}{dq}\frac{\partial q}{\partial x} = 0 \quad \Longrightarrow \quad \frac{w'}{t}\left(\frac{df}{dq} - \frac{x}{t}\right) = 0$$

Possibilities:

Rarefaction fan: $f'[q(x,t)] = x/t$

Shock: $w' = 0 \implies q = \text{constant}$

Why is conservation important?

Shock solution:

$$q(x,t) = \begin{cases} q_L & x < st \\ q_R & x > st \end{cases}$$

What is the shock speed s? Integrate about shock location:

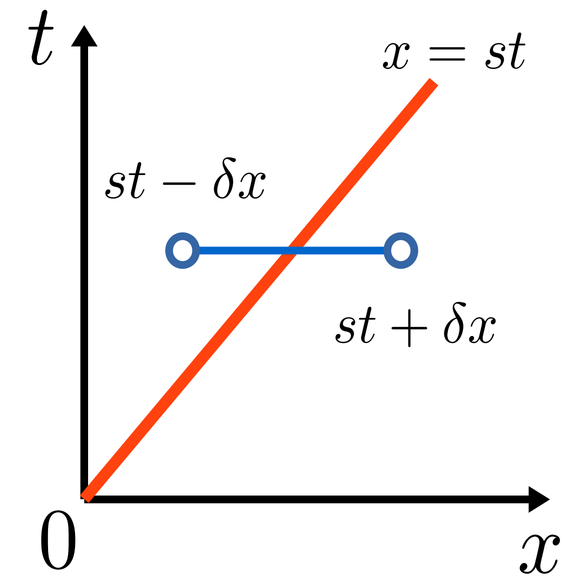
$$\int_{st-\delta x}^{st+\delta x} \frac{\partial q}{\partial t}\, dx = -\int_{st-\delta x}^{st+\delta x} \frac{\partial f}{\partial x}\, dx$$

$$\int_{s-\delta x/t}^{s+\delta x/t} -\frac{\partial q}{\partial \xi} \xi \, d\xi = f(q_L) - f(q_R), \quad \xi \equiv x/t$$

Let $\delta x \to 0$:

$$(q_L - q_R)s = f(q_L) - f(q_R)$$

$$s = \frac{f(q_R) - f(q_L)}{q_R - q_L}$$



If $q_L = q_R$ we just have $s = f'(q_L) = f'(q_R)$

Correct wave speeds require correct jump conditions and thus conservation

Conservation with self-gravity

Gravitating problems have source terms:

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v}\mathbf{v} + P\mathsf{I}) = -\rho \nabla \phi$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot [(\rho E + P)\mathbf{v}] = -\rho \mathbf{v} \cdot \nabla \phi$$

Breaks conservation –

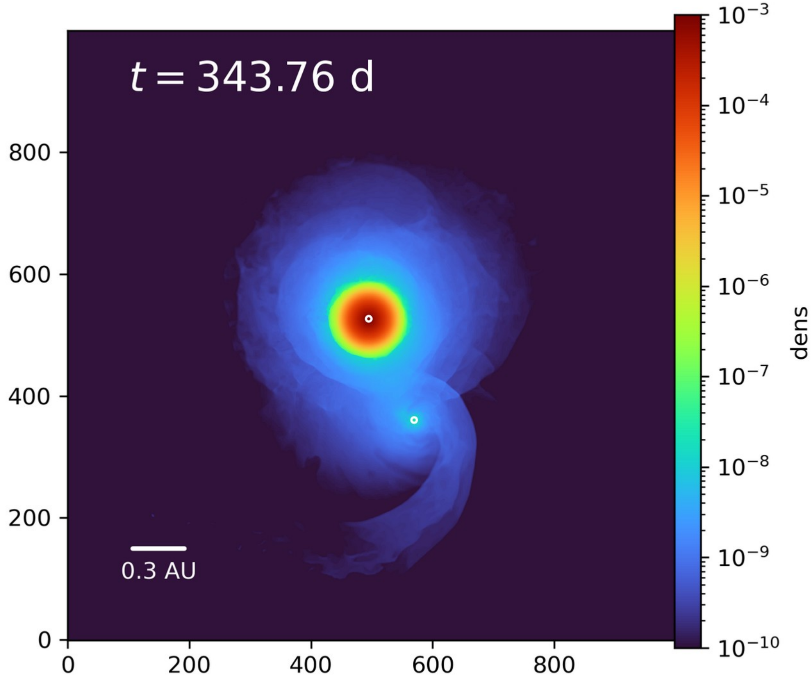
> though self-gravity can be reformulated to conserve energy, with side effects (Jiang et al. 2013)

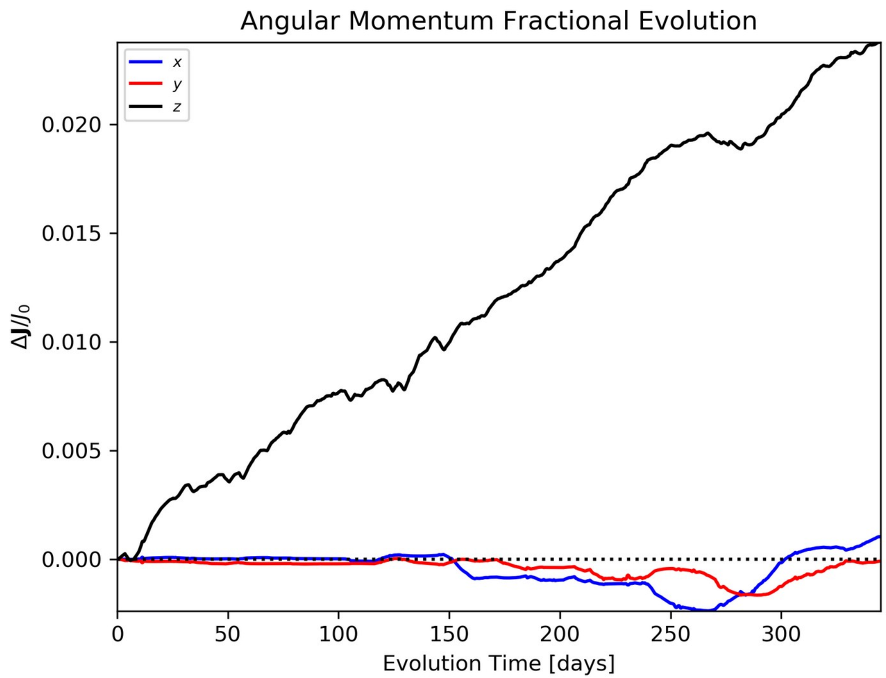
Angular momentum is not conserved in AMR/MM codes, however...

Angular momentum conservation

Zhu et al. (in prep) – low-resolution “wide binary” test – 1.08 + 0.36 system

- Few % change in angular momentum over ~ 4 orbits
- Comparable to final angular momentum for CE case
- Need to vary resolution, fluff properties





Adequate angular momentum conservation is essential for deep inspirals

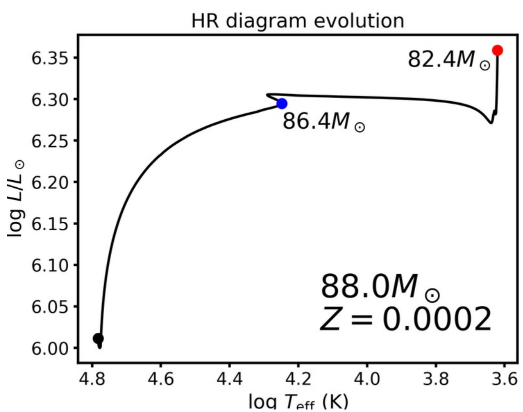
Initial/boundary conditions

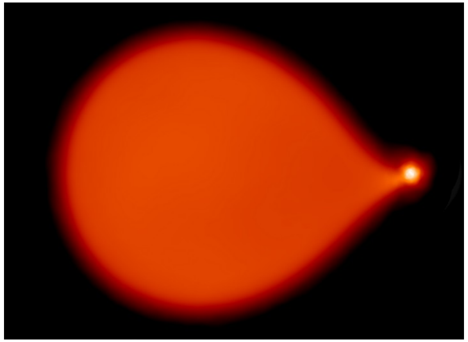
Dealing with disparate timescales

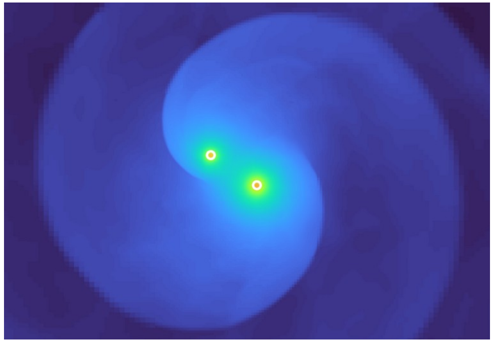
Disparate timescales require a multi-code approach

- Evolve and expand: nuclear ($10^{7\text{-}10}$ yr)
- Lose corotation: tidal (weeks – millennia)
- Plunge: dynamical (months – decades)
- Self-regulated inspiral: thermal (years - millennia)

Back to MESA







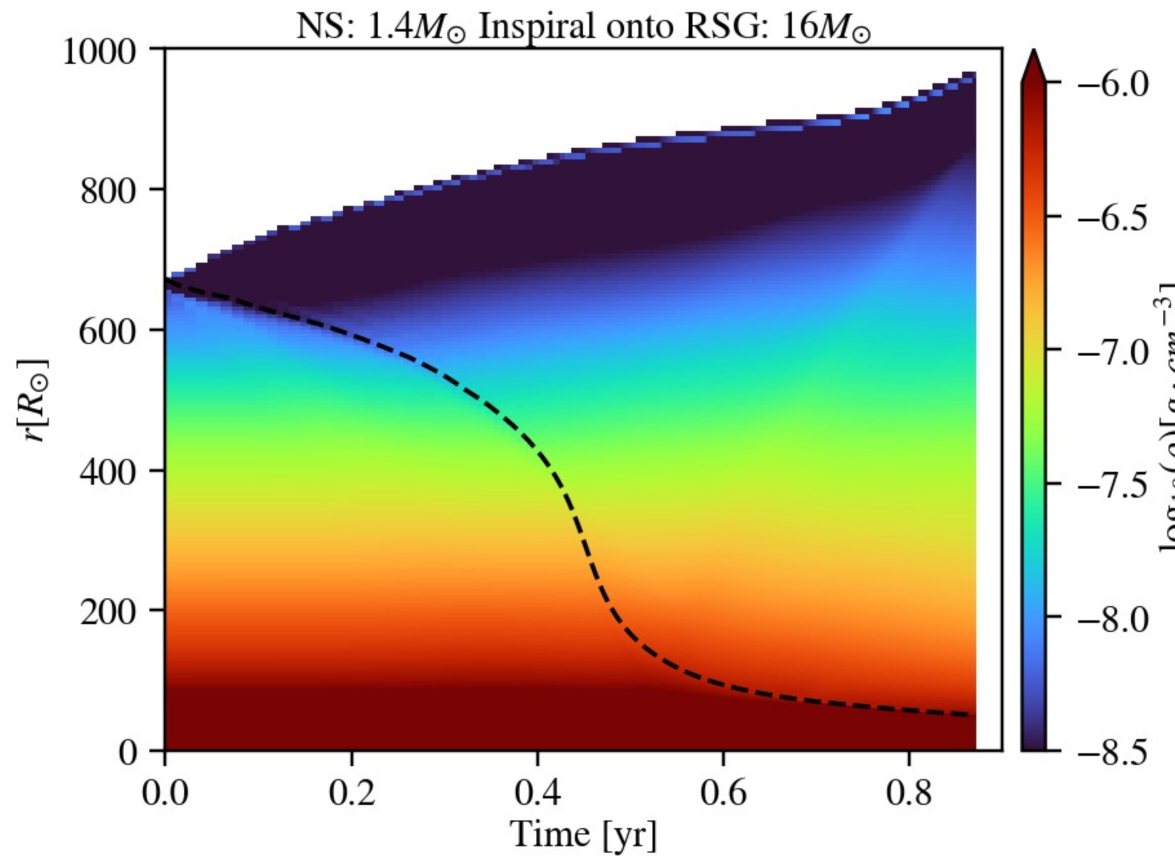
MESA – 1D
(Paxton et al. 2011 – 19)

SPHARG – 3D
(cf Faber & Rasio 2002)

FLASH – 3D
(Fryxell et al. 2000)

Coupling inspiral integration to 1D stellar evolution using MESA

- Drag formalism (MacLeod et al. 2017)
- Understand effect of feedback on donor structure
- With 3D simulations, develop a drag formalism for self-regulated phase
- Use to extend evolution well past the dynamical plunge

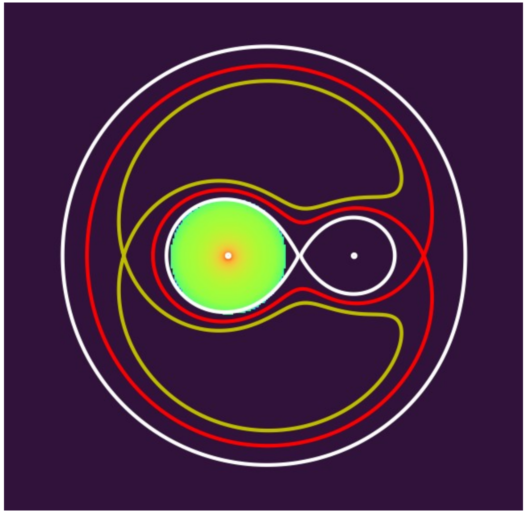


Cruz Lopez et al. (in prep)

Options for initializing 3D simulations from MESA

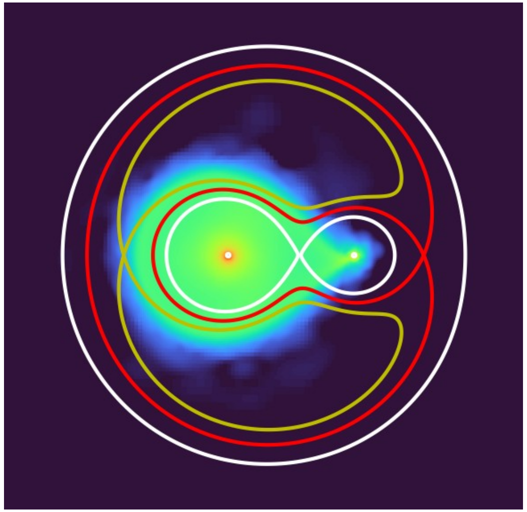
Interpolate directly from 1D model onto 3D code

- Interpolation errors – apply velocity damping
- Total energy of system is not correct
- Donor star is not close to hydrostatic equilibrium



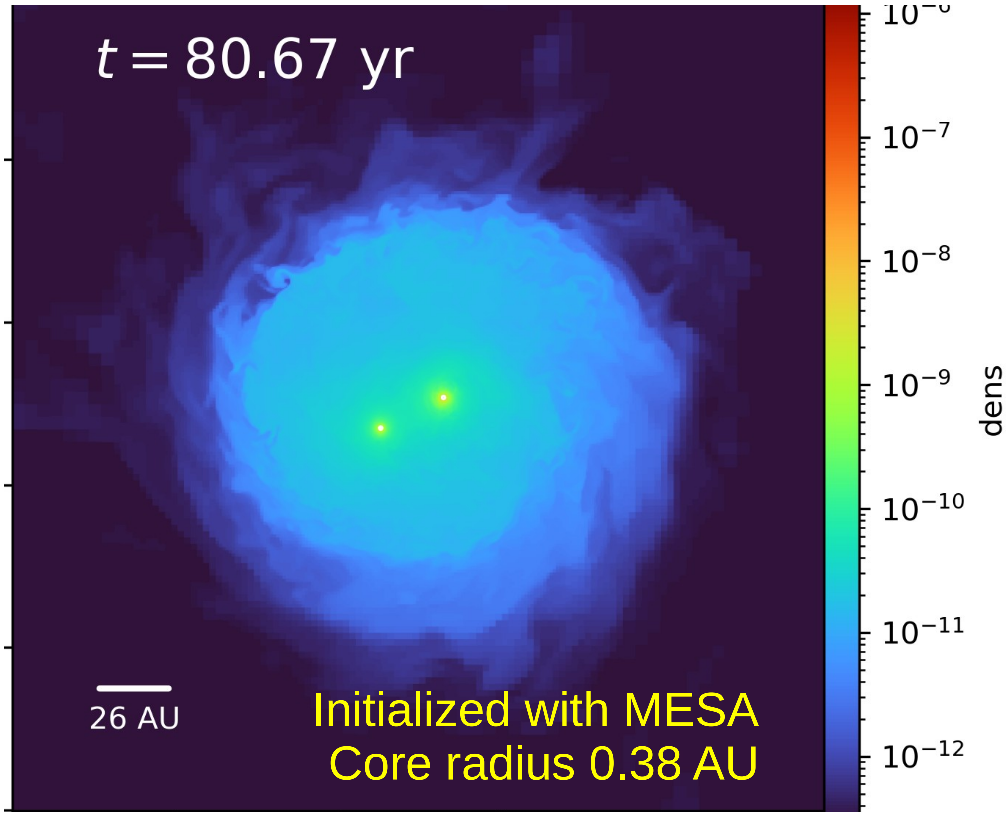
Relax into binary potential

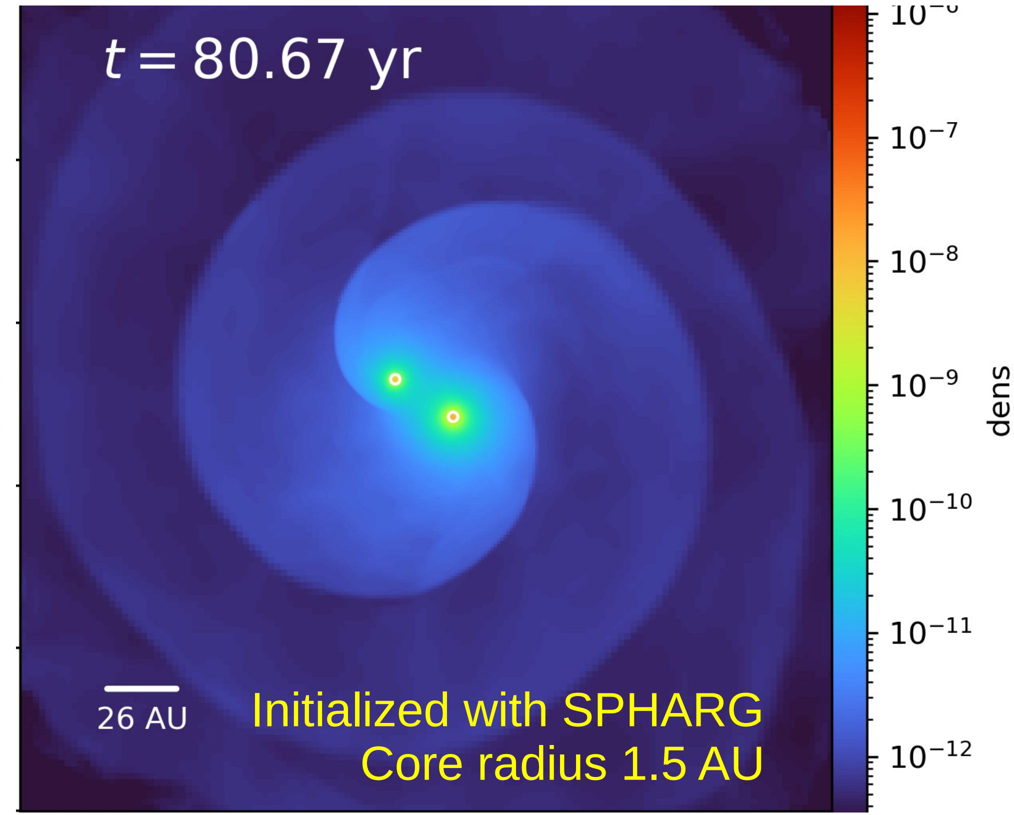
- Start at larger separation – energy more accurate
- Velocity damping to establish HSE
- Evolution timescale too long – so artificially bring stars closer together



Comparing 1D and SPH-relaxed initial conditions

82$M_\odot$, Z=0.0002 RSG + 35$M_\odot$ BH





A relaxed initial model is essential for realistic outcomes

Cores

Can we treat the whole star as gas on the grid?

Not usually: for a red giant donor,

- Required spatial resolution $$\frac{L_{\text{simulation}}}{\Delta x} \sim \frac{a}{r_{\text{core}}} \sim 10^{4-5}$$
- Timestep stability criterion $$\frac{t_{\text{simulation}}}{\Delta t} \sim \frac{t_{\text{orbit}}}{t_{\text{dyn,core}}} \sim \left(\frac{\rho_0}{\bar{\rho}}\right)^{1/2} \sim 10^{7-10}$$

roundoff error!

- Steepness of the core pressure profile – maintaining HSE is hard

➡ usually treat the core as a particle interacting with the gas via gravity only

Using a core particle is a problem if the inspiral takes long enough for the real core to expand/contract

Core subtraction using modified Lane-Emden equation (Ohlmann et al. 2017)

Given MESA model and desired numerical core radius R_{core}:

1. Iteratively solve

$$\frac{1}{r^2}\frac{d}{dr}\left[\frac{r^2}{4\pi G\rho_{\mathrm{rem}}}\frac{dP_{\mathrm{rem}}}{dr} - \frac{r^2}{4\pi G}\frac{GM_{\mathrm{core}}r}{R_{\mathrm{core}}^3}\right] + \rho_{\mathrm{rem}} = 0$$

   in $r \in [0, R_{\mathrm{core}}]$ subject to

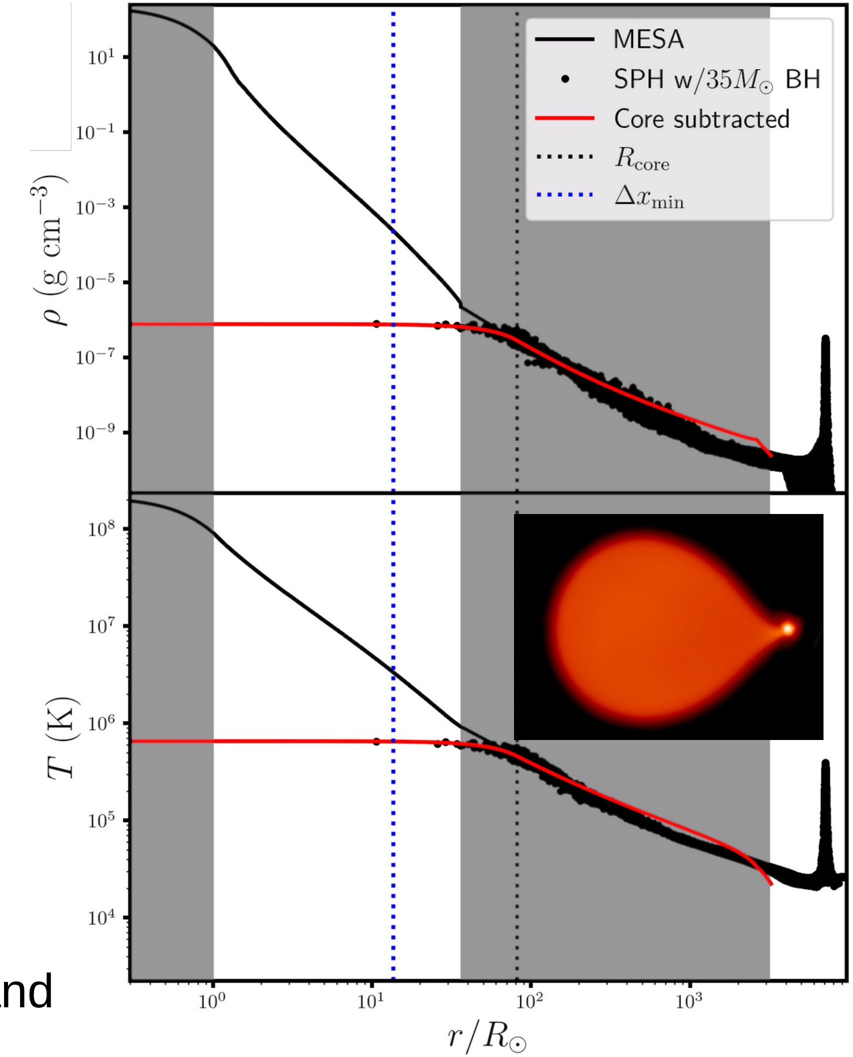
$$M_{\mathrm{core}} + 4\pi\int_0^{R_{\mathrm{core}}} \rho_{\mathrm{rem}} r^2\, dr = m(R_{\mathrm{core}})$$

$$\rho_{\mathrm{rem}}(R_{\mathrm{core}}) = \rho(R_{\mathrm{core}})$$

$$P_{\mathrm{rem}}(r < R_{\mathrm{core}}) = P(R_{\mathrm{core}})\left[\frac{\rho_{\mathrm{rem}}(r)}{\rho(R_{\mathrm{core}})}\right]^{\Gamma_1(R_{\mathrm{core}})}$$

2. Subtract ρ_{rem}, P_{rem} from initial profile and use modified profile to initialize gas

3. Create constant-density core particle with mass M_{core} and radius R_{core}



Fluff

Grid codes require a low-density "fluff" in place of vacuum. Constraints:

- Total mass: should not dominate mass in box or be unstable

→ low density; want $$\frac{\rho_{\rm fluff}}{\bar{\rho}_{\rm star}} \ll \frac{f}{1-f}$$ f = filling factor

- Pressure: should balance surface pressure on star

→ high pressure; want $$\frac{P_{\rm fluff}}{P_{\rm star}} \sim 1 \quad \Longrightarrow \quad \frac{P_{\rm fluff}}{\rho_{\rm fluff}} \gg \frac{P_{\rm star}}{\bar{\rho}_{\rm star}} \quad (f \ll 1)$$

$$\Longrightarrow \quad \frac{kT_{\rm fluff}}{\mu} \gg \frac{GM\,\Delta x}{R^2}$$

- Temperature: should avoid affecting timestep $$\Delta t_{\rm fluff} = \frac{\Delta x}{c_{\rm fluff}} \geq \Delta t_0 = \frac{\Delta x}{c_0}$$

→ low temperature; want $$\Longrightarrow \quad c_{\rm fluff}^2 \leq c_0^2$$

Conflicting requirements!

Fluff with radiation

Fluff is particularly troublesome with radiation (Galaviz et al. 2017)

Dynamics:

- High sound speed in fluff because radiation pressure dominates
- Gas and radiation should decouple, but LTE enforced
- Mixing with high-temperature fluff raises temperature of stellar gas

Predicting light curve and colors:

- T profile steep – hard to determine effective temperature
- Need to artificially reduce opacity of fluff given its density and temperature

Fluff with radiation

Ideas for coping (light curves):

- Make fluff transparent, use $\min(T, T_{\rm eff,init})$ for outermost optically thick zones (Galaviz)
- Fit an atmosphere (Ivanova)

Ideas for coping (dynamics):

- Turn off diffusion below a density threshold
- Nonequilibrium diffusion
- Decoupling gas and radiation below a density threshold
- Photon transport with reduced speed of light

Idea –

Equilibrium diffusion solves gas + radiation energy equation:

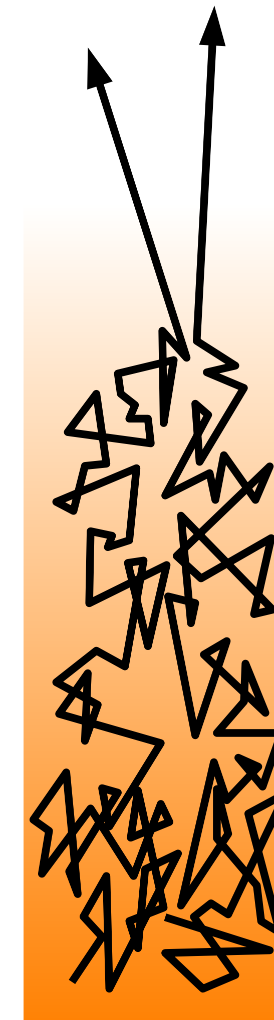
$$\frac{\partial \rho E_{\mathrm{g}}}{\partial t} + \nabla \cdot [(\rho E_{\mathrm{g}} + P_{\mathrm{g}})\mathbf{v}] + \mathbf{v} \cdot \nabla P_{\mathrm{r}} = -(\text{net cooling rate})$$

$$+ \quad \frac{\partial U_{\mathrm{r}}}{\partial t} + \nabla \cdot (U_{\mathrm{r}}\mathbf{v}) + P_{\mathrm{r}} \nabla \cdot \mathbf{v} = (\text{net cooling rate}) - \nabla \cdot \mathbf{F}_{\mathrm{r}}$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot [(\rho E + P)\mathbf{v}] = -\nabla \cdot \mathbf{F}_{\mathrm{r}}$$

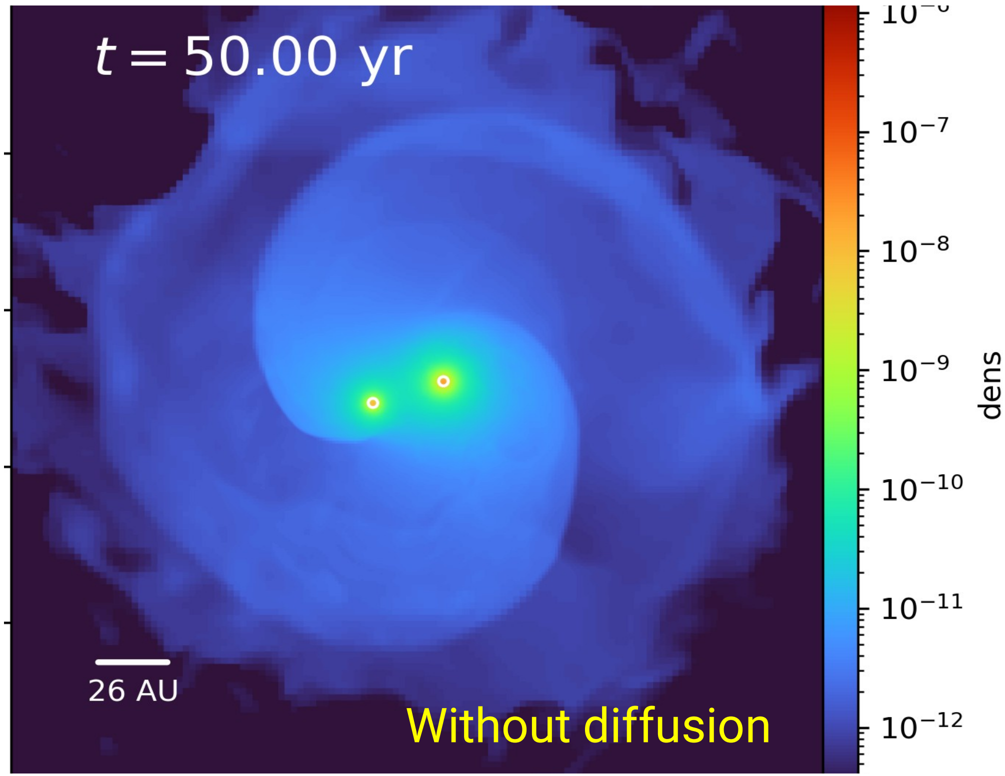
where $E = E_{\mathrm{g}} + \frac{1}{\rho} U_{\mathrm{r}} \quad P = P_{\mathrm{g}} + P_{\mathrm{r}} \quad \mathbf{F}_{\mathrm{r}} = -\frac{c}{\rho\kappa} \nabla P_{\mathrm{r}}$

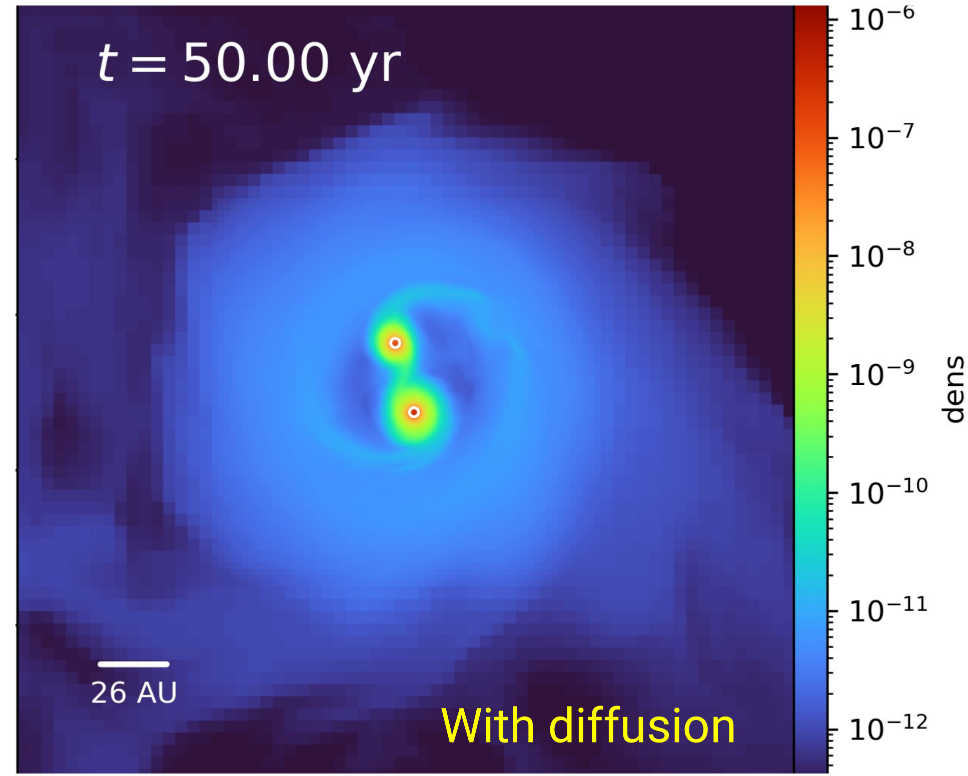
Below $\rho < 10^{-12}$ g cm^{-3}, decouple radiation smoothly: $P_{\mathrm{r}}, |\mathbf{F}_{\mathrm{r}}| \to 0$



82+35 system with and without decoupling diffusion

Core radius: 1.5 AU

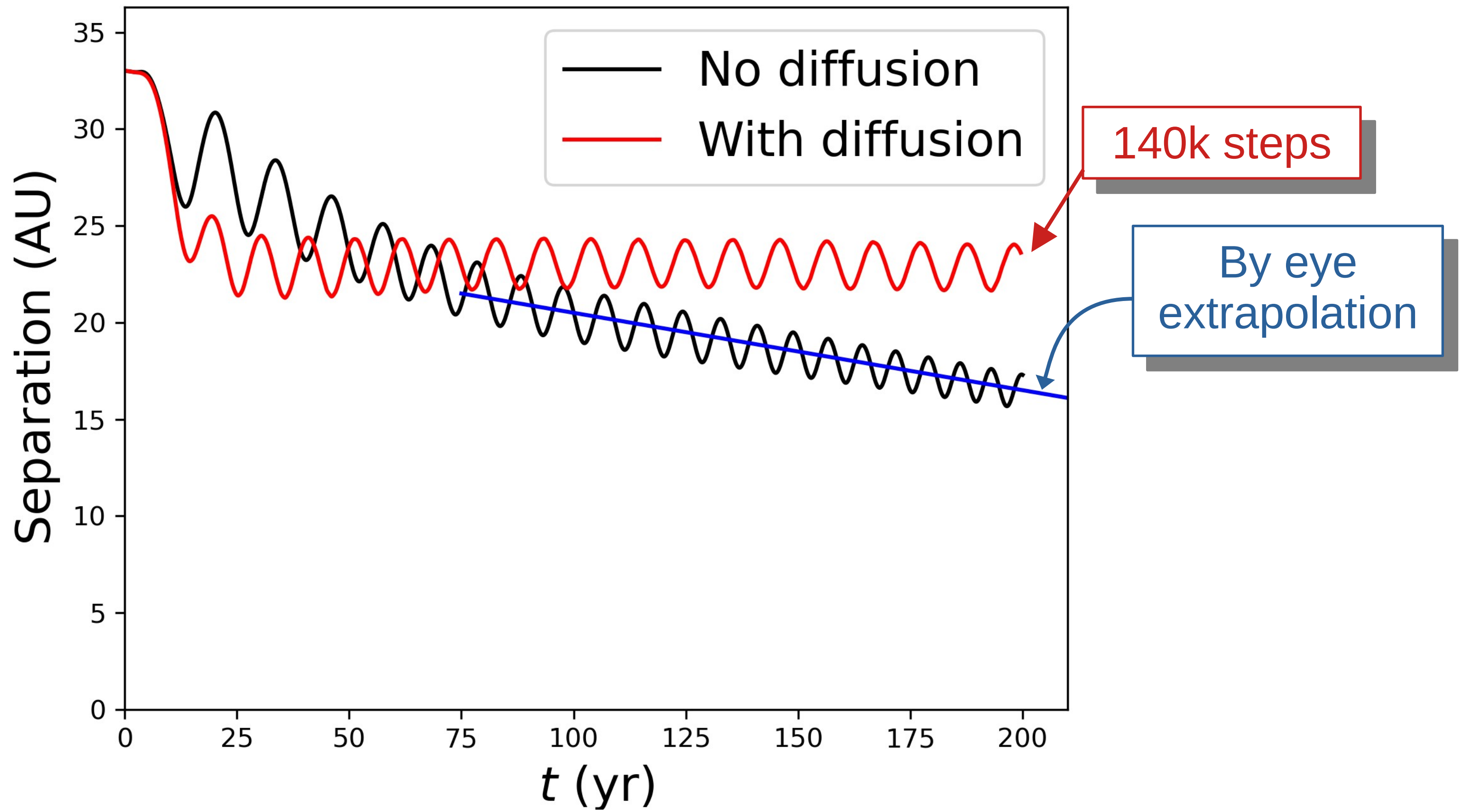




- Without diffusion: inspiral to separation consistent with GW150914 in ~ 600 yr
- With diffusion: weaker shocks, energy losses slow inspiral

82+35 system with and without decoupling diffusion

Core radius: 1.5 AU

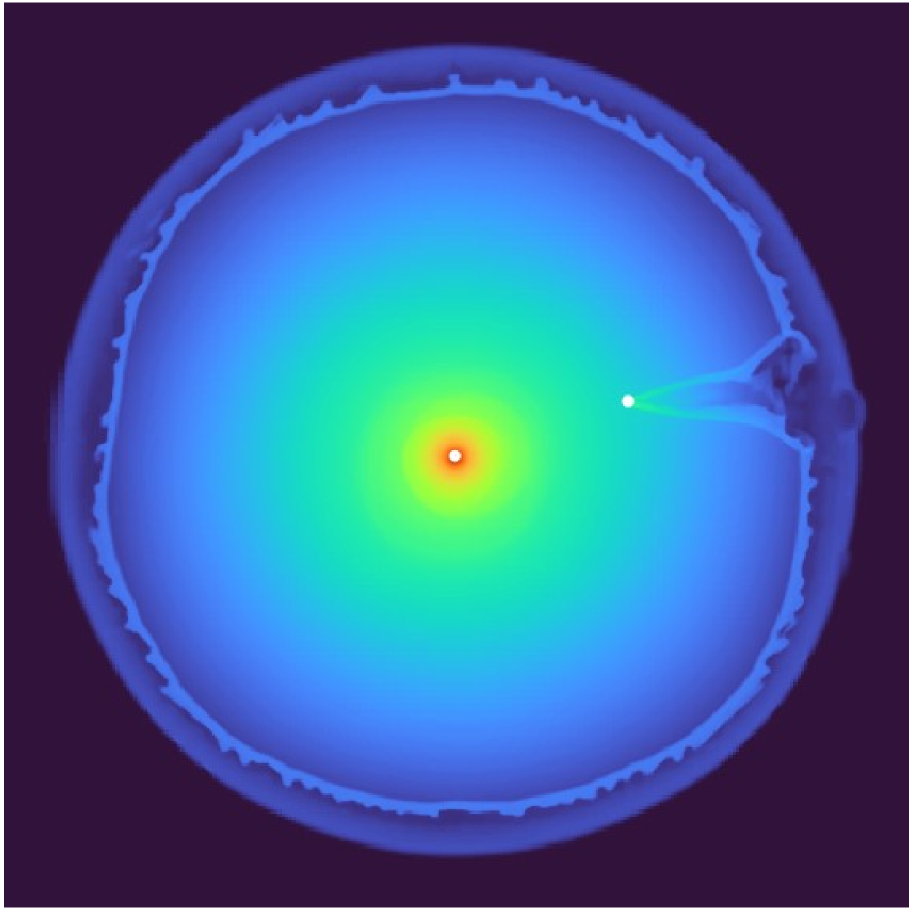


- Without diffusion: inspiral to separation consistent with GW150914 in ~ 600 yr
- With diffusion: weaker shocks, energy losses slow inspiral

Transport can make a big difference for some systems

Fluff matters

A case where the fluff was underpressured



Treatment of the ambient medium must be realistic

Outline

What can we trust?

What can we trust?

Table 4.2. Simplified Comparison of Performance of the Main Types of Simulation Code

Physics	AMR	SPH	MM
Energy conservation	Worst	Best	Worst
Angular momentum conservation	Worst	Best	Intermediate
Galilean invariance	Worst	Best	Better
Advection error	Most	Least	Intermediate
Accurate gravity coupling	Tricky	Easy	Tricky
Vacuum boundary	No, "fluff"	Yes	Yes
Problem geometry	Hardest	Easiest	Easiest
Shock capturing	Best	Worst	Best
Shear instability capturing	Better	Suppressed (old codes), good but slow (modern codes)	Best
Artificial viscosity	Minimal	Significant in shocks	Minimal
Subresolution mixing	Most	Absent by default, but can be added (Read et al. 2010)	Less
Resolution in low-density regions	Easy	No	Easy
Resolution in high-density regions	Harder	Easier	Harder
Radiation hydro	Easiest	Hardest	Intermediate
Magnetohydrodynamics	Easiest	Hardest, but realizations exist (e.g., Price 2012)	Intermediate

Ivanova, Justham, & Ricker (2020)

What do we need?

What we need – my view

Physics

- A proper subgrid turbulence model for convective regions (including high Edd ratio)
- A drag model that can be used at late times with MESA
- Radiation transport/hydro including thick and thin regions
- A better subgrid accretion model
- Dust (CO5BOLD, RAMSES codes)

Numerics

- Better treatment of fluff: real vacuum or real circumstellar medium/wind
- Ideally: get rid of numerical cores; realistically: somehow link to 1D modeling?
- Better angular momentum conservation for AMR/MM codes
- A code comparison project – CE, single-star advection, and wide binary tests